A Fuzzy Controller

for Three Dimensional Line Following of an

Unmanned Autonomous Mobile Robot

A thesis submitted to the

Division of Research and Advanced Studies
of the University of Cincinnati

in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE

in the Department of Mechanical, Industrial, and Nuclear Engineering
of the College of Engineering

1997

by

Nikhil D Kelkar
B. S from Government College of Engineering Pune (India)

Committee Chair : Dr. Ernest Hall

Abstract

Automated Guided Vehicles (AGV's) have many potential applications in manufacturing, medicine, space and defense. The purpose of this thesis is to describe exploratory research on the design of a modular autonomous mobile robot controller. The controller incorporates a fuzzy logic approach for steering and speed control, a neuro-fuzzy approach for ultrasound sensing and an overall expert system for guidance. The advantages of a modular system are related to portability and transportability, i.e. any vehicle can become autonomous with minimal modifications. A mobile robot test-bed has been constructed using a golf cart base. This cart has full speed control with guidance provided by a vision system and obstacle avoidance using ultrasonic sensors. The speed and steering fuzzy logic controller is supervised by a 486 computer through a multi-axis motion controller. The obstacle avoidance system is based on a micro-controller interfaced with ultrasonic transducers. This micro-controller independently handles all timing and distance calculations and sends distance information back to the computer via the serial line. This design yields a portable independent system in which high speed computer communication is not necessary. Vision guidance is accomplished with two CCD cameras which view lines on the left and right of the vehicle. The data is collected by a vision tracking device that transmits the X,Y coordinates of lane marks to the control computer. Simulation and testing of these systems has yielded promising results. This design, in its modularity, creates a portable autonomous fuzzy logic controller applicable to any mobile vehicle with only minor adaptations.

Acknowledgment

First and foremost, I wish to express my deep gratitude to my dissertation advisor, Paul E. Geier Professor of Robotics, Dr. Ernest Hall, for all his valuable guidance, assistance and support throughout my graduate studies at the University of Cincinnati. It was Dr. Hall who introduced me to this field and helped me to understand this technology.

I wish to thank the members of my master's defense committee for their suggestions and support on this research. Their comments on this research are greatly appreciated.

I also gratefully acknowledge the assistance and support of all the graduate and undergraduate students of the Center for Robotics Research. I would like to thank Tayib Samu for his support in this research.

My parents have encouraged me throughout my education and career, and I will always be grateful for their sacrifice, generosity and love.

Most importantly I extend my gratitude to my wife Dipti for her support, patience and assurance during my pursuit for higher studies.

Table of Contents

List of Tables

List of Figures

Chapter 1. Introduction

Controller design for any system needs some knowledge about the system. Usually this involves a mathematical description of the relation among inputs to the process, its state variables, and its output. This description is called the model of the system. The model can be represented as a set of transfer functions for time invariant linear systems or other relationships for non-linear or variant systems.

Modeling complex systems can be a very difficult task. In a complex system such as a multiple input and multiple output system, inaccurate models can lead to unstable systems, or unsuitable system performance. Fuzzy Logic Control (FLC) is an effective alternative approach for systems which are difficult to model. The FLC uses the qualitative aspects of the human decision process to construct the control algorithm. This can lead to a robust controller design. The modeling of a mobile robot is a very complex task and a direct application of FLC can be found in this area.

An excellent introduction to the mathematical analysis of mobile robots can be found in a study by Muik and Neuman [1]. Even though the visualization and recognition of image information for the guidance of mobile robot have been studied for years, [2-7] the design of a mobile system is a challenging task. The challenge of designing an intelligent controller is in determining what information to measure and how to use this information in a manner that will satisfy the performance specifications of the machine. Several fuzzy logic approaches for modeling control systems for vehicles have been studied in the past. A fuzzy logic controller [8] that guarantees stability of a control system for a computer-

simulated model car and advanced fuzzy logic application for automobiles application was discussed in Altrock et. al. [9].

Objective

The main aspect of intelligent control addressed in this thesis is the design of a controller for a mobile robot using fuzzy logic. The design specifications were to build a robot which could follow a line, avoid obstacles, and adapt to variations in terrain. The adaptive capabilities of a mobile robot depend on the fundamental analytical and architectural designs of the sensor systems used. The mobile robot provides an excellent test bed for investigations into generic vision guided robot control since it is similar to an automobile and is a multi-input, multi-output system. An algorithm was developed to establish a mathematical and geometrical relationship between the physical three dimensional (3-D) ground coordinates of the line to follow and its corresponding two dimensional (2-D) digitized image coordinates. This relationship is incorporated into the vision tracking system to determine the perpendicular distance and angle of the line with respect to the centroid of the robot. The information from the vision tracking system was used as input to a closed loop fuzzy logic controller to control the steering and the speed of the robot.

Fuzzy Logic

History, Research & Applications

Fuzzy logic was first proposed by Lotfi A. Zadeh of the University of California at Berkeley in a 1965 paper; he elaborated on his ideas in a 1973 paper [10] that introduced the concept of "linguistic variables". In his approach, a fuzzy implication is defined as a fuzzy cartesian product. Based on Zadeh's definition of implication, Mamdani [11] built the first fuzzy logic controller.

Other implications have been studied by Baldwin and Guild [12] and Baldwin and Pilsworth [13]. They have proposed the following properties of implication: fundamental property, smoothness property, unrestricted inferences, modus ponens, modus tollens symmetry, and propagation of fuzziness.

Research concerning applicability of various implications in fuzzy model has been conducted by Kiszka [14] and then by Cao and Kandel [15]. Using linguistic descriptions of a d.c series motor, Kiszka constructed its fuzzy model using 36 fuzzy implications and two different interpretations of connective ALSO. He developed a root-mean-square error, as well as the number of mathematical operations, required to obtain the fuzzy model, such as a criterion of applicability given a fuzzy implication. Cao extended the investigation to different physical systems, with different fuzzy sets used to interpret the linguistic terms.

In 1995 Maytag introduced an "intelligent" dishwasher based on a fuzzy controller and a "one-stop sensing module" that combines a thermistor (for temperature

measurement), a conductivity sensor (to measure detergent level from the ions present in the wash), a turbidity sensor that measures scattered and transmitted light to measure the soiling of the wash, and a magnetostrictive sensor to read spin rate. The system determines the optimum wash cycle for any load to obtain the best results with the least amount of energy, detergent, and water; it even adjusts for dried-on foods by tracking the last time the door was opened and estimates the number of dishes by the number of times the door was opened.

For the last 20 years, Fuzzy Control Theory has emerged as a fruitful approach to a wide variety of control problems [16,17,18,19,20,21,22,24].

During an international meeting of fuzzy logic researchers in Tokyo, Takeshi Yamakawa [24] demonstrated the use of fuzzy control (through a set of simple dedicated fuzzy logic chips) in an "inverted pendulum" experiment -- a classic control problem in which a vehicle tries to keep a pole (mounted on its top) by a hinge upright by moving back and forth. Observers were impressed with this demonstration, as well as later experiments by Yamakawa in which he mounted a wine glass containing water or even a live mouse to the top of the pendulum; the system maintained stability in both cases. Yamakawa eventually went on to organize his own fuzzy-systems research lab to help develop his patents in the field.

A comparison of Fuzzy Logic, Proportional Integrated Derivative (PID) and Sliding Mode Control is presented by Coleman and Godbole [25]. They conclude that Fuzzy Logic is a very robust control tool for a control engineer. In their paper [25] they present a comparison of Fuzzy Logic Control and Classical Control Design

methodologies. The Fuzzy Logic Control achieved all the goals that the PID and sliding mode control is normally used to achieve.

Zhou, McLauchlan, Challoo and Omar [26] have presented a Fuzzy Logic Control of a four-link robotic manipulator in a vertical plane. A non-algorithmic, model free approach has been developed that relies on a fuzzy rule base to evaluate the required axis motion to result in the desired two-dimensional end point motion. This scheme does not require solution of complex nonlinear inverse kinematics equations to arrive at the joint control commands. This is in contrast with the traditional robot controllers. The fuzzy rule base control provided a fast execution speed.

Research and development is also continuing on fuzzy applications in software (as opposed to firmware) design, including fuzzy expert systems and integration of fuzzy logic with neural-network and evolutionary adaptive "genetic" software systems, with the ultimate goal of building "self-learning" fuzzy control systems; however, this subject is beyond the scope of this thesis.

Research Objective

The main goal of this research is to model a modular Fuzzy Logic Control for an automated guided vehicle and test the performance of the vehicle in a real time environment. The research is focused on the design of the Fuzzy Controller for vision and sonar navigation of the automated guided vehicle.

The design of the controller was done in three stages.

In the *first stage* the universe of discourse was identified and fuzzy sets are defined. The rule base (Fuzzy Control Rules) for the control was defined through a human decision making process. The membership functions and their intervals are defined. Aggregation and defuzzification methods are selected.

In the *second stage* the Fuzzy Controller was coded in C programming language and implemented on the autonomous guided vehicle. Then the performance of the controller was then tested through a series of simulations and real time running of the vehicle.

In the *third* and final stage, the rule base and the membership functions were tuned to improve the performance of the vehicle.

Outline of the thesis

The fundamentals of the fuzzy set theory and fuzzy logic theory are presented in Chapter 2. The discussion includes basic definitions and concepts used in this research.

Chapter 3 contains a detailed description on the Fuzzy Logic Controller. The algorithm used in the research is described in detail. The concept of Binary Fuzzy Associative Memory is presented.

Chapter 4 has a brief introduction to the Autonomous Guided Vehicle Project "Bearcat" and the international Ground Robotics Competition.

Chapter 5 deals with data input for vision navigation and the details of the fuzzy sets, rule base, membership functions and implication operators.

Chapter 6 presents the results of simulation runs using MATLAB with detail a detail review of a few selected cases. It also contains a small summary of the real time testing on the robot.

Finally the conclusions for this research are presented in Chapter 7. The simulation programs and the source code for the fuzzy controller are listed in the appendices.

Chapter 2. Theory of Fuzzy Sets

Fuzzy set and fuzzy logic theories are reviewed in this chapter. These theories are the basis of fuzzy logic control. The definitions and concepts presented in this chapter are based on Novak and Klir [28] and Folger [28]. The information is repeated in this chapter for completeness and to introduce and clarify the notation used in the rest of this thesis.

The fuzzy set is essentially a generalization of the classical or ordinary set. The ordinary set is defined in such a way that individuals in some given universe of discourse are divided into two groups: members - those that certainly belong in the set and non-members - those that certainly do not. A sharp unambiguous distinction exists between the members and non-members of the class or category represented by the ordinary set. Many of the categories commonly employed to describe our perception of reality do not exhibit this sharp distinction. For example, there is no sharp distinction between the classes of luxury cars, moderately expensive cars and economy cars. Instead, their boundaries are vague, and the transition from member to non-member is gradual rather than abrupt. Thus, the fuzzy set introduces vagueness by eliminating the sharp boundaries dividing members of the class from non-members. Mathematically, a fuzzy set is defined by assigning to each individual in the universe of discourse a value in the interval [0, 1] representing its grade of membership in the fuzzy set. This grade corresponds to the degree to which that individual is similar to or compatible with the concept represented by the fuzzy set. Thus, individuals may belong in a fuzzy set to a

greater or lesser degree indicated by a larger or smaller membership grade. Full membership and full non-membership are indicated by membership grade of 1 and 0, respectively.

Basic concepts

Definitions

Definition 1

Let C be an ordinary set called the universe of discourse that may be either discrete or continuous and let U be the function taking values from the interval [0,1]. The fuzzy subset C in the universe of discourse C maybe represented by the function:

$$U : C \longrightarrow [0,1] \tag{1}$$

where U is called the membership function of fuzzy set C:

Thus the fuzzy subset $\overline{C}$ in C is represented as a set of ordered pairs of an element c and its membership function:

$$C = \{(U - (c) / c) | c \equiv C\} \tag{2}$$

When a membership function takes only the values 1 or 0, the fuzzy set becomes the ordinary set. The value of the membership function for a given element of a fuzzy set is called the membership grade.

Definition 2

The support of fuzzy set $\overline{C}$ is an ordinary set defined as:

$$su(\overline{C}) = \{c : U(c) \neq 0\} \tag{3}$$

Definition 3

The α-cut of the fuzzy set $\overline{C}$ for the given α is an ordinary set:

$$\overline{C}(\alpha) = \{c : U(c) \geq \alpha\} \quad (4)$$

Definition 4

A fuzzy singleton is a fuzzy set whose support is a single point in C with U =1.

Example: Let X will be a universe of discourse defined as X= { 0, 1,2,3,4,5} and let LV be the fuzzy subset defined on X corresponding to 'x is large'.

LV={0 / 0 , 0 / 1 , 0. 1 / 2 , 0.3 / 3 , 0.8 / 4 , 1 / 5} then:

su (L-V) = {2,3,4,5}

LV (α =O.3) = {3,4,5}

Two things should be noted. First, based on definition 1, the membership function is equivalent to the fuzzy set determined by this membership function (Novak, 1989). Second, fuzzy sets should not be regarded as a some kind of probability. Although probabilities, as well as membership functions, take values from the closed interval [0,1], and both concepts are used to express uncertainty, there is a significant difference between these two concepts. One obvious distinction is that the summation of probabilities on a finite universal set must equal 1, but there is no such requirement for membership grades.

Operations on fuzzy sets

The three basic operations on fuzzy sets are: complement, union, and intersection. The basic operations are defined as functions that satisfy certain axiomatic requirements and operate on membership grades of fuzzy sets. The results of these operations are membership functions of new fuzzy sets representing the concept of fuzzy complement, union, and intersection.

Fuzzy complement

In order for any function com : $[0, 1] \rightarrow [0, 1]$ to be defined as a fuzzy complement, the following requirements need to be fulfilled:

Axiom 1 For any fuzzy compliment com (com(0))= 1 and com(l)=0

Axiom 2 For all a, b $\equiv$ [0, 1], if a < b then com(a) > com(b)

In most cases of practical significance, the class of function that can be defined as fuzzy complement must be narrowed. Thus, two additional axioms are introduced:

Axiom 3 com is a continuous function.

Axiom 4 com is involute, which means that com(com(a)) = a for all a $\equiv$ [0, 1].

Axiom 1 assures that fuzzy complement does not violate the complement for ordinary sets. Moreover an increase in the degree of membership in a fuzzy set should cause a decrease in the degree of membership of its complement. This is provided by Axiom 2. Axioms 1 and 2 are called the axiomatic skeleton for fuzzy complement. The basic complement operator is given below:

For example let

LV = {0 / 0 , 0 / 1 , 0. 1 / 2 , 0.3 / 3 , 0.8 / 4 , 1 / 5}

be a fuzzy set then, the fuzzy complement is given as

-LV = {0 / 0 , 1 / 1 , 0. 9 / 2 , 0.7 / 3 , 0.2 / 4 , 0 / 5}

Fuzzy union

The fuzzy union is in general a function U of the form u : [0, 1] x [0, 1] $\rightarrow$ [0, 1]. For each element x in the universal set, this function takes as its argument the pair consisting of the element's membership grades in set A and in set B and yields the membership grade of the element in the set constituting the union of A and B. Thus:

$$U(x) = U\left[U_A, U_B(x)\right] \quad (5)$$

where U(x) is membership function of fuzzy union A $\cup$ B

This function has to satisfy the following requirements:

Axiom 1 U(0,0) = 0 and U(0,1) = U(1,O) = u(l,l) = 1

Axiom 2 U(a,b) = U(b,a).

Axiom 3 If a $\leq$ b and c $\leq$ d, then U(a,c) $\leq$ U(b,d) (U is monotonic)

Axiom 4 U(U(a,b),c) = U(a,U(b,c)) (U is associative)

The first axiom ensures that the fuzzy union is a generalization of the classical union for ordinary sets. This axiom is analogous to the first axiom for fuzzy compliment. The second axiom indicates indifference to the order in which the sets are combined in fuzzy union. The third axiom is the natural requirement that a decrease in the degree of membership in fuzzy sets can not produce an increase in the degree of membership of their union. Finally, the fourth axiom allows one to extend the fuzzy union operation for any number of fuzzy sets. As used in the discussion of the fuzzy complement two additional axioms are formulated. These are:

Axiom 5 U is continuous function.

Axiom 6 U(a,a) = a

The axiom of continuity prevents a situation, where a small change in the membership grade in fuzzy sets produces a large change in the membership grade in their union. Axiom 6 ensures that the union of any fuzzy set with itself yields precisely the same set. Based on the above definitions, many union operators can be formulated. The most important and most used in approximate reasoning are:

Max union	$a \cup b = \max(a, b)$	
Algebraic sum	$a \cup b = a + b - ab$	
Bounded sum	$a \cup b = \min(l, a + b)$	
Drastic sum	$a \cup b = a$	when $b = 0$
	$a \cup b = b$	when $a = 0$
	$a \cup b = 0$	when $a, b \geq 0$
Disjoint sum	$a \cup b = \max(\min(a, 1 - b), \min(l - a, b))$	

Fuzzy intersection

The fuzzy intersection is defined as a function i such that 1:

[0, 1] x [0, 1] $\rightarrow$ [0, 1]. For each element x in the universal set, this function takes as its argument the pair consisting of the element's membership grades in set i and in set B,and yields the membership grade of the element in the set constituting the union of A and B. Thus:

$$U(x) = i\left[U_A, U_B(x)\right] \qquad (6)$$

where U(x) is membership function of fuzzy union A $\cap$ B

This function has to satisfy the following conditions:

Axiom 1 i(0,0) = 0 and i(0, 1) = i(1,0) = i(l, 1) = 1

Axiom 2 i(a,b) = i(b,a).

Axiom 3 If a $\leq$ b and c $\leq$ d, then i(a,c) i(b,d) $\leq$ (U is monotonic)

Axiom 4 i(i(a,b),c) = i(a,i(b,c)) (i is associative)

Axiom 5 i is continuous function.

Axiom 6 i(a,a) = a

The fulfillment of the two last axioms is unnecessary. The motivation for the above conditions is the same as for fuzzy union. Moreover, all axioms, except the first, are identical to axioms formulated for fuzzy union. The first axiom states that fuzzy intersection has to reduce to ordinary set intersection when the grade of membership is restricted to 0 and 1. There is an infinite number of functions that can be used as fuzzy

intersection, however only a few of them have been employed in approximate reasoning. These are:

Min Intersection	$a \cap b = \min(a, b)$	
Algebraic product	$a \cap b = - ab$	
Bounded product	$a \cap b = \max(0, a + b - 1)$	
Drastic product	$a \cap b = a$	when $b = 1$
	$a \cap b = b$	when $a = 1$
	$a \cap b = 0$	when $a,b < 1$

Aggregation operator

Aggregation operations on fuzzy sets are operations by which several fuzzy sets are combined to produce a single set. In general, any aggregation operation may be defined by the function

$h: [0,1]^n \rightarrow [0,1]$ (7)

where n is a number of aggregated fuzzy sets

This function has to satisfy the following axiomatic requirements:

Axiom 1 h(0,0,.....,0) = 0 and h (1,l,.....,1) = 1 (boundary conditions).

Axiom 2 h is monotonic non-decreasing in all its arguments.

Therefore fuzzy unions and intersections qualify as aggregation operations on fuzzy sets. They can be viewed as special aggregation operations that are symmetric, usually continuous, and required to satisfy some additional boundary conditions. The simplest aggregation operators are:

- -min. intersection - ^

- max. union - V

Fuzzy Logic (FL)

The natural consequence of fuzzy set theory is fuzzy logic. The logic connectives AND and OR are defined based on fuzzy set operations. Fuzzy AND corresponds to fuzzy intersection operator and fuzzy OR to fuzzy union.

In Boolean logic that is based on ordinary set theory, one can have only two logic values. Any element can be only in one of two possible relations to the set: it can belong to the set or not belong. Therefore something can be only true or false. It can be seen easily that this logic is unrealistic. Usually it is difficult to absolutely state that something is true of false. More often one simply does not know, or knows subjectively.

For example the logic value of a statement:

"Jimmy is tall" /and Jimmy is 5'10"/ can not be evaluated by Boolean logic.

Such problems are nonexistent in fuzzy logic. Fuzzy sets introduce intermediate logic values. Now, logical value can be any number within the interval [0, 1]. Values 0 and 1 mean that something is absolutely false or absolutely true. If the logic value of a given statement is somewhere between 0 and 1, for example 0.7, it means that this statement is close to the absolute truth with strength 0.7 or, that this is more true than false. Therefore, fuzzy logic is more flexible than Boolean logic and closer to reality.

Fuzzy implication operators

Fuzzy implication is one of the crucial elements in FLC. These implication operators interpret the information in linguistic statements and present it in a form to facilitate decision making by the FL controller. Every control rule of the FL controller rule base is expressed as a fuzzy relation.

The fuzzy implication is defined as a combination of fuzzy conjunction - fuzzy logic operator AND, fuzzy disjunction - fuzzy logic OR, and fuzzy negation. These operators were defined in the previous section in terms of operations on fuzzy sets. The correspondences among the logic operators and fuzzy set operators are shown below:

FUZZY SET THEORY	FUZZY LOGIC
fuzzy intersection i	fuzzy conjunction AND
fuzzy union u	fuzzy disjunction OR
fuzzy complement com	fuzzy negation NOT

Example Implications:

Mamdani implication :a=>b=min(a,b) is just fuzzy conjunction defined by the min intersection operator.

Chapter 3. Fuzzy logic controller

The Fuzzy Logic Controller uses the fuzzy set and fuzzy logic theory previously introduced in its implementation. A detailed reference on how to design a fuzzy controller can be found in [29,30,31].

Fuzzy Inference System

Fuzzy inference is the actual process of mapping from a given input to an output using fuzzy logic [27]. Fuzzy logic starts with the concept of a fuzzy set. A fuzzy set is a set without a crisp, clearly defined boundary. It can contain elements with only a partial degree of membership. The MATLAB Fuzzy Logic Toolbox was used to build the initial experimental fuzzy inference system.

In Figure 1 is a schematic of the control strategy for the automated guided vehicle.

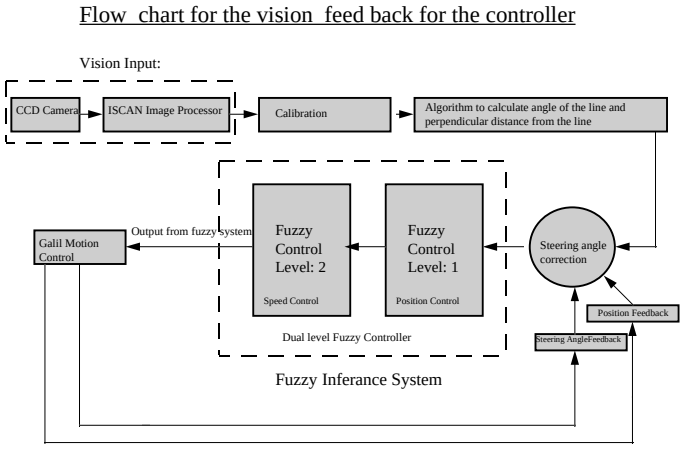


Figure 1. Flow chart of vision navigation feed back

The fuzzy inference process consists of following steps:

- Fuzzy Set Definition
- Fuzzification
- Rule Base
- Fuzzy Operators
- Implication
- Aggregation
- Defuzzification

Fuzzy Set Definition

The universe of discourse is identified and fuzzy sets are defined. The definition of these sets requires expert knowledge of the control system. The most common shape of membership functions is triangular, although trapezoidal and bell curves are also used, but the shape is generally less important than the number of curves and their placement. From three to seven curves are generally appropriate to cover the required range of an input value (universe of discourse).

The shape of the membership function is also determined at this step. In this application a triangular membership function is used. The reason behind this is that they are easier to represent and implement, as a result the complexity of the problem is reduced. The fuzzy set definition is more important than the shape of the membership function.

Input fuzzy sets

The first level of the fuzzy system has two inputs, θ_{error} and d_{error}. These inputs are resolved into a number of different fuzzy linguistic sets. For θ_{error}, there are seven fuzzy sets. The scale for this input is -20 to 20

Fuzzy Set	Description
a-3	Extreme left
a-2	Left
a-1	Soft left
a	Zero
a1	Soft right
a2	Right
a3	Extreme right

Table 1 Input fuzzy sets

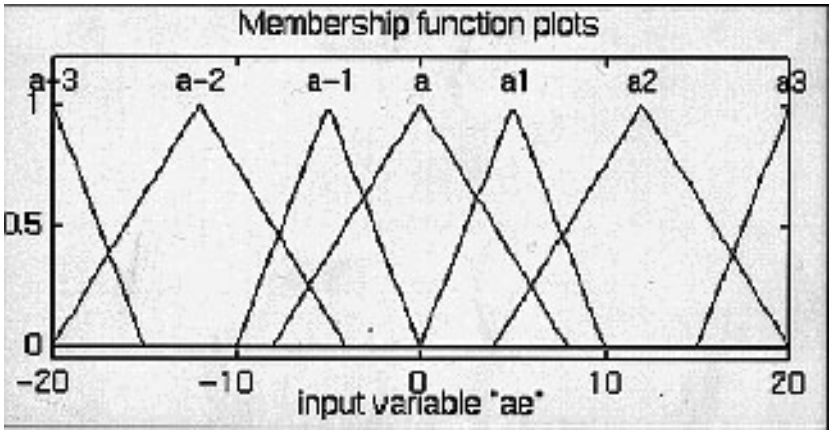


Figure 2. Input fuzzy sets (angle error)

Figure 2 shows the input membership functions for angle error ae:

while d_{error} also has five sets: Scale for this input is -30 inches to 30 inches

Fuzzy Set	Description
d-2	Far left
d-1	Left
d	Zero
d1	Right
d2	Far right

Table 2 Input fuzzy sets

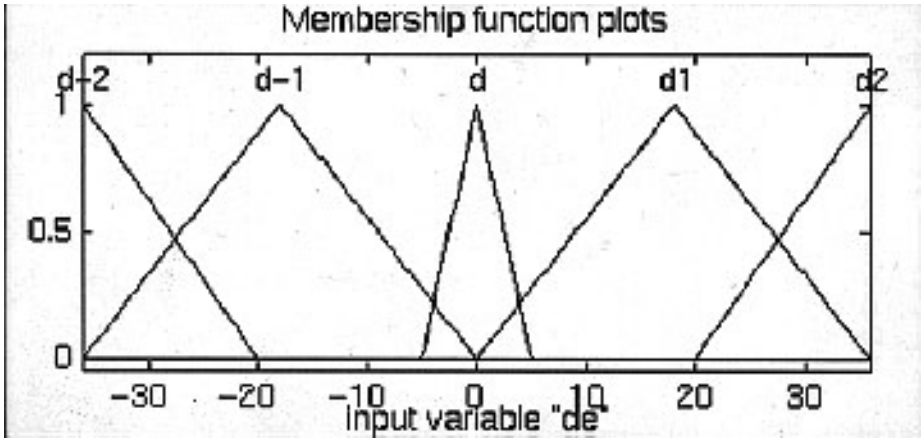


Figure 3. Input fuzzy sets (distance error)

Output fuzzy set

The output of the first level of the fuzzy system is $\theta_{steering}$. It has seven fuzzy linguistic sets. $\theta_{steering}$ is on a scale of -20 to +20 degrees. The output fuzzy sets are:

Fuzzy Set	Description
BP	Big positive
MP	Medium positive
SP	Small positive
ZE	Zero
SN	Small negative
MN	Medium negative
BN	Big negative

Table 3 Output fuzzy sets

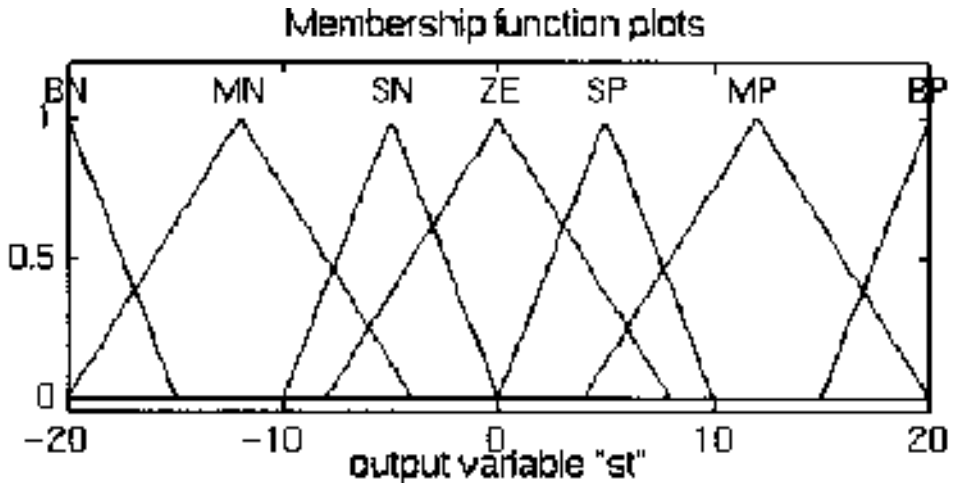


Figure 4. Output fuzzy sets (corrected steering angle)

Figure 4 shows the output membership functions for steering angle correction st:

Fuzzification

The input data (crisp data) is fuzzified according to the membership functions. The fuzzified input is the degree to which each part of the antecedent has been satisfied for each rule.

Rule Base

The way one develops control rules depends on whether or not the process can be controlled by a human operator. If the operator's knowledge and experience can be explained in words, then linguistic rules can be written immediately. If the operator's skill can keep the process under control, but this skill cannot be easily expressed in words, then control rules may be formulated based on the observation of operator's actions in terms of the input - output operating data. However, when the process is exceedingly complex, it may not be controllable by a human expert. In this case, a fuzzy model of the process is built and the control rules are derived theoretically. It should be noted however, that this approach is quite complicated and has not yet been fully developed. Therefore the FLC is ideal for complex ill - defined systems that can be controlled by a skilled human operator without the knowledge of their underlying dynamics. In such cases an FL controller is quite easy to design and implementation is less time consuming than for a conventional controller.

Determination of control rules remains a problem. Generally there are three possible methods described as follows.

1. The method based on a fuzzy model of the process.

The relation between inputs and outputs can be expressed using linguistic descriptions. This description is referred to as the fuzzy model of the system and takes the form of conditional statements. Based on this model the optimal set of control rules can be obtained [32].

2. The method based on the operator's experience and/or the control engineer's knowledge.

These control rules are similar to rules formed by the human decision process. This case is used in formulation of rules for the purpose of this controller. This makes the rule formulation straightforward. In a man - machine interaction rules can also be formed according to the operator's control actions.

3. The method based on learning

A FLC can be developed which can learn from a good or a bad experience. Such a controller was first proposed by Mamdani in 1979.

In this case the rules were expressed linguistically with the help of human logical decision making process. These rules can be appended along the way as the human expertise increases.

Following are the rules used for the control expressed as IF THEN rules:

1. IF (de is d-2) AND (ae is a-3) THEN (st is BN)

2. IF (de is d-2) AND (ae is a-2) THEN (st is BN)

3. IF (de is d-2) AND (ae is a-1) THEN (st is MN)

4. IF (de is d-2) AND (ae is a) THEN (st is SN)

5. IF (de is d-2) AND (ae is a1) THEN (st is SN)

6. IF (de is d-2) AND (ae is a2) THEN (st is SN)

7. IF (de is d-2) AND (ae is a3) THEN st is SN)

8. IF (de is d-1) AND (ae is a-3) THEN (st is BN)

9. IF (de is d-1) AND (ae is a-2) THEN (st is MN)

10. IF (de is d-1) AND (ae is a-1) THEN (st is MN)

11. IF (de is d-1) AND (ae is a) THEN (st is SN)

12. IF (de is d-1) AND (ae is a1) THEN (st is SN)

13. IF (de is d-1) AND (ae is a2) THEN (st is SP)

14. IF (de is d-1) AND (ae is a3) THEN (st is SP)

15. IF (de is d) AND (ae is a-3) THEN (st is MN)

16. IF (de is d) AND (ae is a-2) THEN (st is MN)

17. IF (de is d) AND (ae is a-1) THEN (st is SN)

18. IF (de is d) AND (ae is a) THEN (st is ZE)

19. IF (de is d) AND (ae is a1) THEN (st is SP)

20. IF (de is d) AND (ae is a2) THEN (st is MP)

21. IF (de is d) AND (ae is a3) THEN (st is MP)

22. IF (de is d1) AND (ae is a-3) THEN (st is SN)

23. IF (de is d1) AND (ae is a-2) THEN (st is SN)

24. IF (de is d1) AND (ae is a-1) THEN (st is SP)

25. IF (de is d1) AND (ae is a) THEN (st is SP)

26. IF (de is d1) AND (ae is a1) THEN (st is MP)

27. IF (de is d1) AND (ae is a-3) THEN (st is SP)

28. IF (de is d2) AND (ae is a-2) THEN (st is SP)

29. IF (de is d2) AND (ae is a) THEN (st is SP)

30. IF (de is d2) AND (ae is a1) THEN (st is MP)

31. IF (de is d2) AND (ae is a2) THEN (st is BP)

32. IF (de is d2) AND (ae is a3) THEN (st is BP)

Fuzzy Operators

The fuzzy operator is applied to the fuzzified input and results in a single number that represents the result of the antecedent for that rule. This number can be

then applied to the output membership function. The inputs to the fuzzy operator are two or more membership values from fuzzified input variables.

In the controller presented we use a AND operator (product).

Implication

The implication method is defined as shaping of the consequent (a fuzzy set) based on the antecedent (a single number). The input for the implication process is a single number given by the antecedent, and the output is a fuzzy set.

The controller presented in this research uses an AND (product) implication which scales the output fuzzy set.

Aggregation

Aggregation is the method for unifying the outputs of each rule by joining the parallel threads. In this process all the output fuzzy sets from each rule are united into one fuzzy set.

The SUM (simple sum of each rule's output) type of aggregation is used in this controller.

Defuzzification

The input for defuzzification is the fuzzy set (the aggregate output fuzzy set) and the output is a crisp number. Defuzzification can be obtained using three known ways:

- The mean of maximum method
- The maximizing decision
- The center of gravity method

Centroidal defuzzification method[28] (center of gravity method) is perhaps the most popular method is used for defuzzification.

$$U_0 = \frac{\sum_{j=1}^{n} U(U_j) U_j}{\sum_{j=1}^{n} U(U_j)} \tag{8}$$

U_0 is determined by means of a gravity center of the area under the membership function curve of the fuzzy output and $U(U_j)$ is a membership grade of U_j

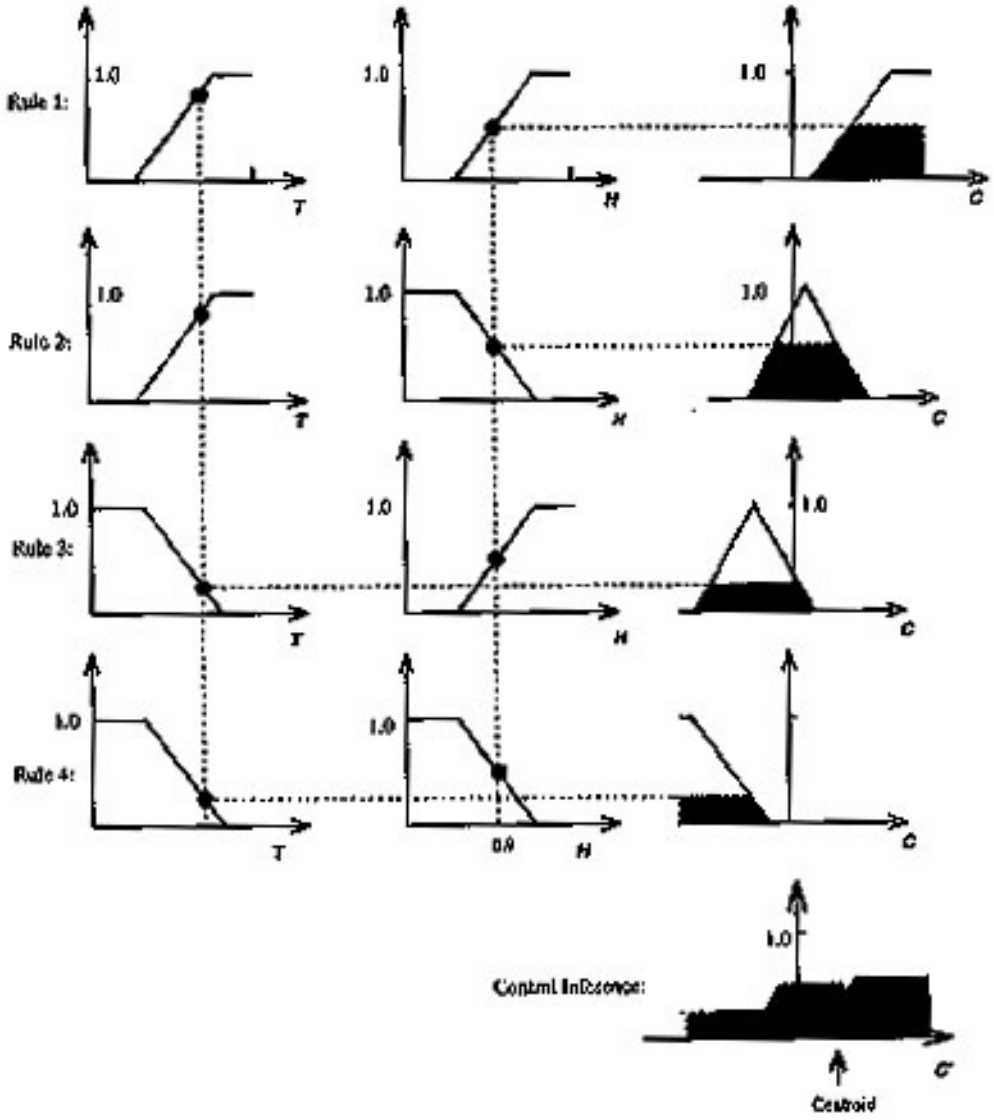


Figure 5. Fuzzy centroidal defuzzification

Figure 5 shows how the fuzzy implication works and the center of gravity defuzzification.

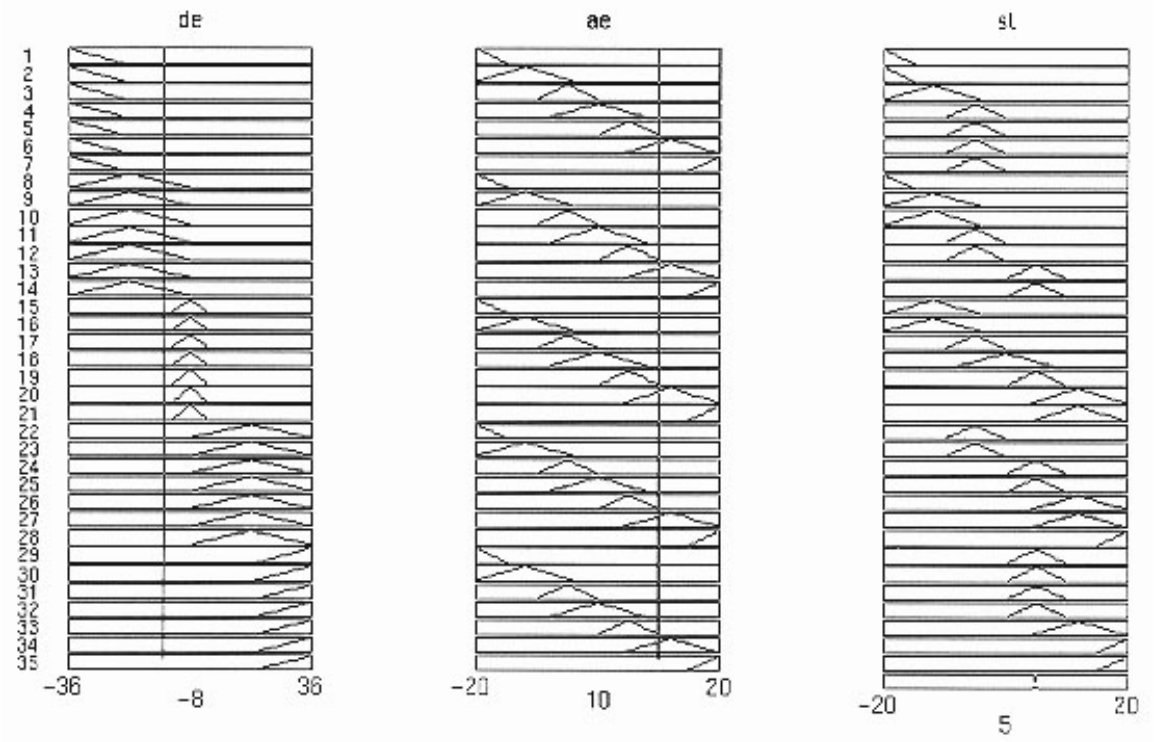


Figure 6. Fuzzy rule base for a particular input

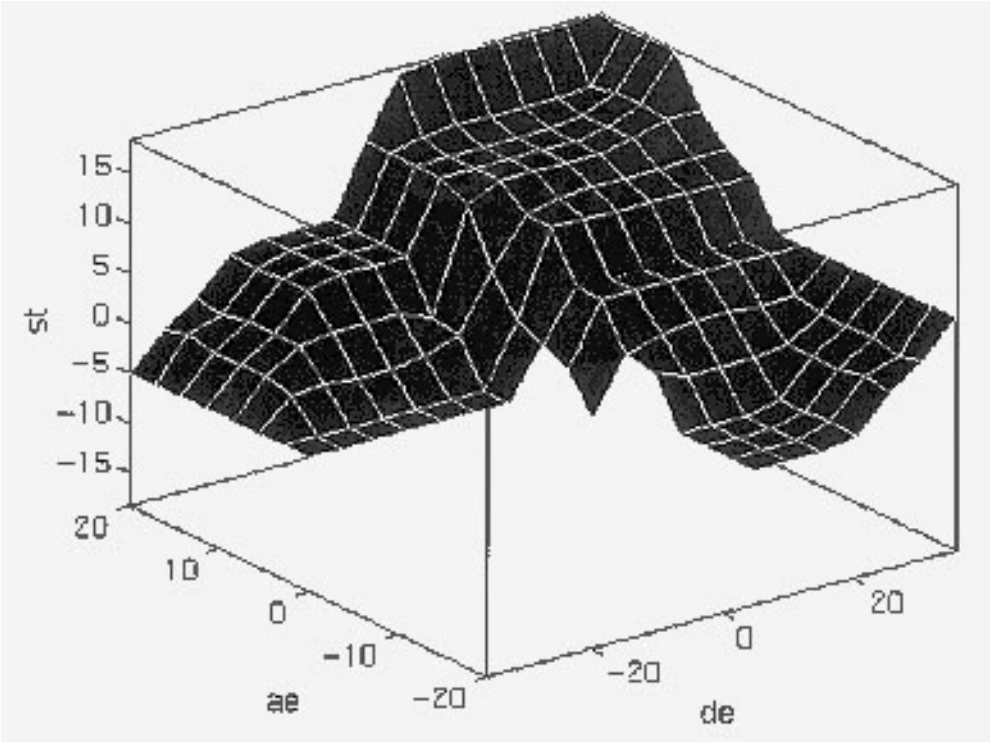


Figure 7. Rule surface for the fuzzy controller

Disadvantages of FLC

Although FLC provides the tools to incorporate human expert knowledge and experience in automatic control systems, there are two important disadvantages:

Because a FL controller is a model-free estimator, it is difficult to develop a model that can prove the stability of a fuzzy control system. One solution is to use conventional control systems in conjunction with fuzzy systems to ensure stability.

Tuning of an FL controller is still an open problem for which no known analytical technique presently exists. Therefore, tuning must be accomplished by a trial and error procedure.

Chapter 4. About the competition and AGV design

Each year for the past five years Oakland University hosts the AUVS International Ground Robotics Competition. The goal is to build an automated guided robot that can navigate around an outdoor obstacle course. The competition is for undergraduate and/or graduate student teams. Key event sponsors include the Association for Unmanned Vehicle Systems International, the Society of Automotive Engineers, the US Army TARDEC, the National Automated Highway Systems and the Oakland University in Rochester Michigan. Figure 8 illustrates the UC Robot Club robot BEARCAT in action in the competition.

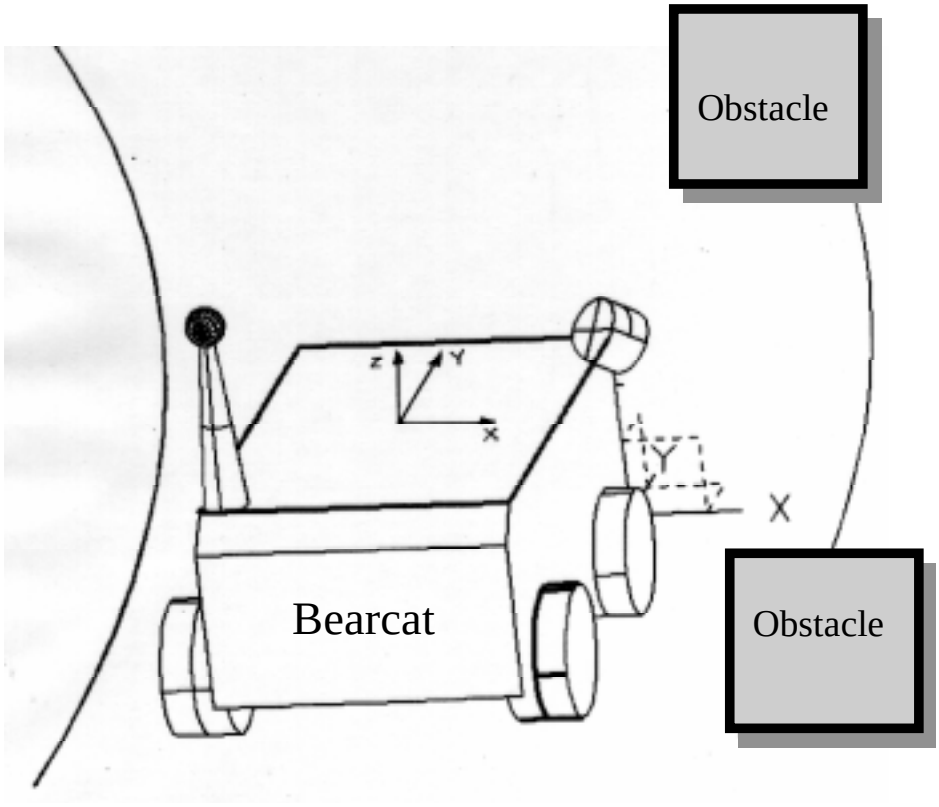


Figure 8. UC Bearcat on the obstacle course

Objective

The objective of the competition is to promote research for automated guided vehicles which can navigate (vision and sonar navigation) in between two lines (continuous and dashed) on the ground. These vehicles should also have the capacity to detect and avoids obstacles in their path.

Environment

Figure 9 is a schematic of the robot running in its environment. The UC Robot "Bearcat" uses two CCD cameras for vision navigation. The Bearcat also uses sonar based obstacle detection and avoidance. A complete design description can be found in[33].

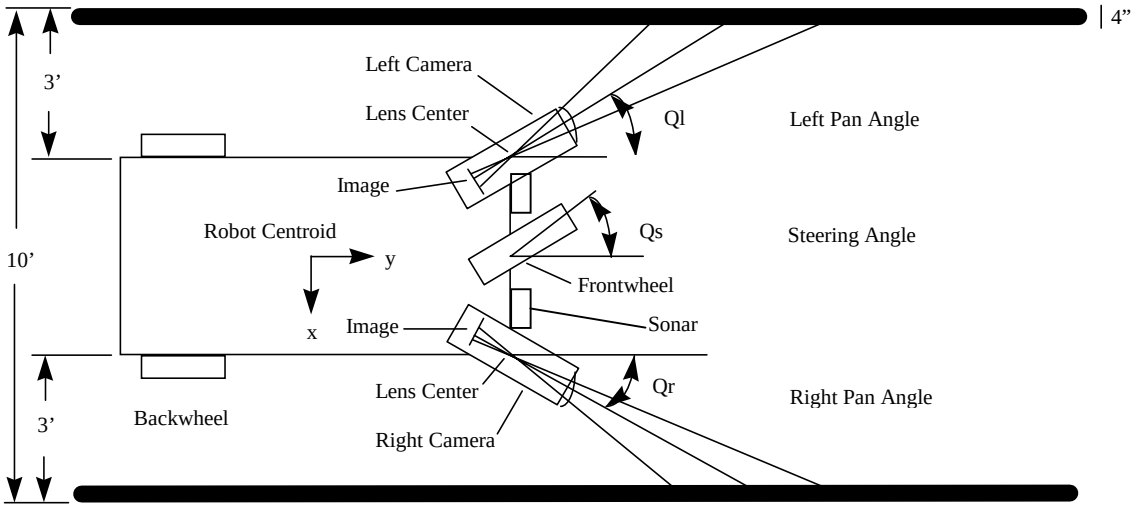


Figure 9. Environment of the robot during line following

Chapter 5. Control

Vision guidance and data input

The purpose of the vision guidance is to guide the robot to follow the line using a digital CCD camera. To do this, the camera needs to be calibrated. Camera calibration [34] is a process to determine the relationship between a given 3-D coordinate system (world coordinates) and the 2-D image plane a camera perceives (image coordinates). More specifically, it is to determine the camera and lens model parameters that govern the mathematical or geometrical transformation from world coordinates to image coordinates based on the known 3-D control field and its image. The CCD camera digitizes the line from 3-D coordinate system to 2-D image system. Since the process is autonomous, the relationship between the 2-D system and the 3-D system has to be accurately determined so that the robot can follow the line. The objective of this section is to show how a model was developed to calibrate the vision system so that, given any 2-D image coordinate point, the system can mathematically compute the corresponding ground coordinate point. The X and Y (the Z is constant) coordinates of two ground points are then computed from which the angle and the perpendicular distance of the line with respect to the centroid of the robot are determined.

The vision system was modeled by the following equations.

$$xPI = A_{11} x_g + A_{12} y_g + A_{13} z_g + A_{14} \quad (9)$$

$$yPI = A_{21} x_g + A_{22} y_g + A_{23} z_g + A_{24} \quad (10)$$

where A_{nm} are coefficients, xPI and yPI are x and y image coordinates, and x_g, y_g, and z_g are the ground coordinates. In transforming the ground coordinate points to the image coordinate points the following transformation operations occur on the points: scaling, translation, rotation, perspective, and projective. Solving for the transformation parameters to obtain the image and ground coordinate relationship is a difficult task. Fortunately, in the model equations given above, the transformation parameters are imbedded into the coefficients. To compute the coefficients, a calibration device was built to obtain 12 data points. With the 12 points, a matrix equation was yielded as shown below:

$$XPI = CA_{1k} \tag{11}$$

$$YPI = CA_{2k} \tag{12}$$

where

$$C = \begin{bmatrix} xg_1 & yg_1 & zg_1 & 1 \\ xg_2 & yg_2 & zg_2 & 1 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ xg_{12} & yg_{12} & zg_{12} & 1 \end{bmatrix}; XPI = \begin{bmatrix} xPI_1 \\ xPI_2 \\ \cdot \\ \cdot \\ \cdot \\ xPI_{12} \end{bmatrix}; YPI = \begin{bmatrix} yPI_1 \\ yPI_2 \\ \cdot \\ \cdot \\ \cdot \\ yPI_{12} \end{bmatrix}; A_{1k} = \begin{bmatrix} A_{11} \\ A_{12} \\ A_{13} \\ A_{14} \end{bmatrix}; A_{2k} = \begin{bmatrix} A_{21} \\ A_{22} \\ A_{23} \\ A_{24} \end{bmatrix}$$

Equations (11) and (12) consist of 12 linearly independent equations and four unknowns, the least-square regression method is applied to yield a minimum mean-square error solution for the coefficients. Below are the equations for the solution:

$$A_{1K} = (C^TC)^{-1}C^TXPI \tag{13}$$

$$A_{2K} = (C^TC)^{-1}C^TYPI \tag{14}$$

Given an image coordinate xPI and yPI, and z ground coordinate (the z coordinate of the points with respect to the centroid of the robot is maintained constant since the robot is run on a flat surface in this model) the corresponding x_g and y_g ground coordinates are computed as indicated by the following matrix equations.

$$\begin{pmatrix} x_g \\ y_g \end{pmatrix} = Q^{-1}\mathrm{B} \tag{15}$$

where

$$Q = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}; B = \begin{bmatrix} xPI - A_{14} - (A_{13} z_g) \\ yPI - A_{24} - (A_{23} z_g) \end{bmatrix}$$

Note that equation (9) and (15) can be modified to accommodate the computation of z_g when an elevation of the ground surface is considered.

The image processing of the physical points is done by the ISCAN tracking device, which returns the centroid of the brightest or darkest region in a computer controlled windows and returns its X and Y coordinates. Two points on the line are windowed and their corresponding coordinates are computed as described above. From the computed x and y ground coordinates of the points, the angle of the line with respect to the centroid of the robot is computed from simple trigonometric relationship. In the next section, we shall show how the angle of the line just computed is used with other parameters to model the steering control of the robot with a fuzzy logic controller.

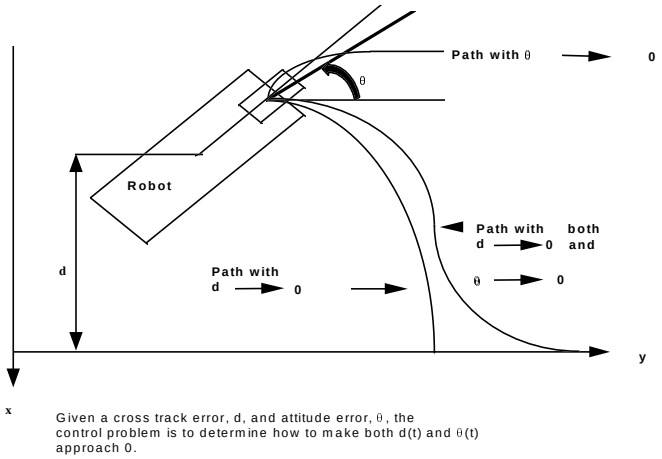


Figure 10. Line following

Obstacle Avoidance

The obstacle avoidance system consists of six ultrasonic transducers. A ultrasonic ranging system from Polaroid is used for the purpose of calibrating the ultrasonic transducers. An Intel 80C196 microprocessor and a circuit board with a liquid crystal display is used for processing the distance calculations. The distance value is returned through a RS232 port to the control computer. The system requires a isolated power supply: 10-30 VDC, 0.5 amps.

The two major components of an ultrasonic ranging system are the transducer and the drive electronics. In the operations of the system, a pulse of electronic sound is transmitted toward the target and the resulting echo is detected. The elapsed time between the start of the transit pulse and the reception of the echo pulse is measured. Knowing the speed of sound in air, the system can convert the elapsed time into a distance measurement.

The drive electronics has two major categories - digital and analog. The digital electronics generate the ultrasonic frequency. A drive frequency of 16 pluses per second at 52 kHz is used in this application. All the digital functions are generated by the Intel microprocessor. The analog functionality is provided by the Polaroid integrated circuit. The operating parameters such as the transmit frequency, pulse width, blanking time and the amplifier gain are controlled by polakit, a developers software provided by Polaroid.

Pseudo code for obstacle avoidance

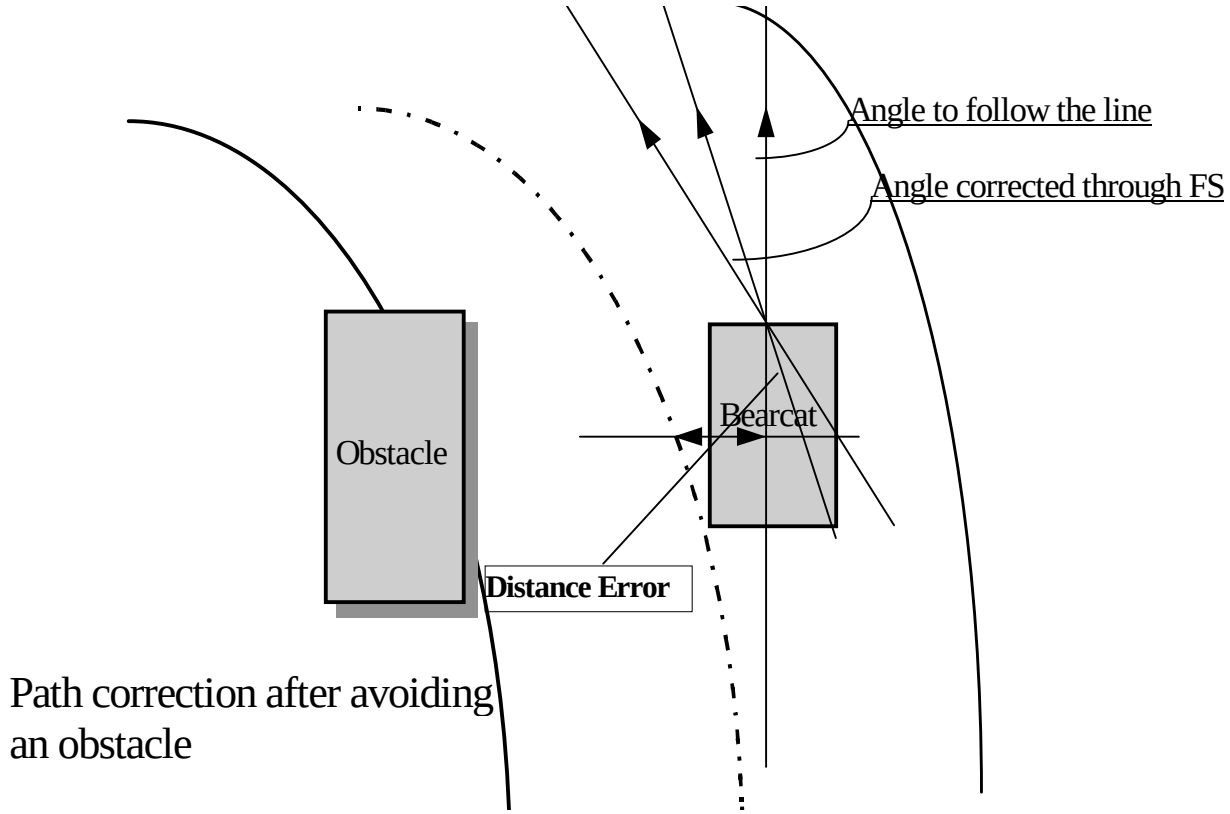


Figure 11. Obstacle avoidance strategy

Figure 11 presents the strategy used for obstacle avoidance. The pseudo code presented explains how the obstacle avoidance is used in the control algorithm.

Loop starts

Get input from vision input (Scanning device)

Process and validate input.

Calculate the distance error and the angle error.

Check for obstacle on the right and left

If the obstacle is less than three feet and greater than two feet

Change the distance error value

If the distance error is negative and the obstacle is on the right

Increase the distance error by 10 inches

If the distance error is negative and the obstacle is on the left

Reduce the distance error by 5 inches

If the distance error is positive and the obstacle is on the right

Reduce the distance error by 5 inches

If the distance error is positive and the obstacle is on the left

Increase the distance error by 10 inches

Input the distance error and angle error to the fuzzy controller

Steer the robot according to the output steering angle.

If the obstacle is less than two feet and greater than one feet

If the obstacle is on the left steer the robot to right by 10 degrees

Else steer the robot to the left by 10 degrees

If the obstacle is within one feet

    If the obstacle is on the left steer the robot to right by 20 degrees

    Else steer the robot to the left by 20 degrees

End Loop

The strategy for obstacle avoidance, in short, is to modify the distance error input to the fuzzy controller and use the same fuzzy controller again to compute the corrected motion direction. This way redesign of a separate controller for obstacle avoidance is avoided. The pseudo code explains how and by what amount the distance error should be modified.

Chapter 6. Results

The testing of the dual level fuzzy system controller explained in Chapter 3 was done in two steps. First, a theoretical simulation was run using the MATLAB fuzzy tool box. The input for this simulation was generated using theoretical test cases. The inputs to the MATLAB batch file which was used to run the simulation (M-file) were θ_{error} and d_{error}. The outputs were $\theta_{steering}$ and the speed of the robot. Seventeen cases were considered. Results of the simulation are shown in Table 1 . Appendix C has the code for the M-file. Note that the results appear reasonable for both angle and speed. The results of these simulations encouraged further tests using a real life scenario. As a result the model was also implemented on the mobile robot, which is scheduled to take part in a nation wide obstacle avoidance and path following competition in June '97.

Remarks:	Distance Error:(de)	Angle Error: (ae)	Steering angle: (st)	Speed of the robot: (speed)
Case 1:Ideal	0	0	0	3.5591
Case 2:	2	0	-5.2	3.2751
Case 3:Extreme	3	0	-10	2.5057
Case 4:Extreme	-3	0	10	2.5057
Case 5:Extreme	0	20	19.6	1.4679
Case 6:Extreme	2.5	20	-19.6	1.4679
Case 7:Extreme	-3	20	19.6	1.4679
Case 8:	-1.5	-1	10	2.5057
Case 9:	1.5	10	-13.6	1.9084
Case 10:	-2	13	10.8	2.356
Case 11:	-2.8	5	13.6	1.9084
Case 12:	1.8	-20	-19.6	1.4679
Case 13:	-2.2	-15	10.8	2.356
Case 14:	2.6	15	-19.2	1.4811
Case 15:	0	-12	-10	2.5057
Case 16:Extreme	3	19	-19.6	1.4679
Case 17:	1.2	-8	-13.6	1.9084

Table 4. MATLAB simulation results (theoretical)

Matlab Simulations

Simulations were run using artificially made-up and real-time data collected using the robots vision system and the ultrasonic obstacle avoidance system. Five cases were considered for this study. The collected data was used as an input to the fuzzy inference system implemented in C programming language and MATLAB was used to provide graphics support. Each case from case one to five was made more difficult for the controller by introducing noise in the input data in form of input data variations caused by loss of vision (loss of track of line) and obstacles in the path.

Case 1: Straight Line

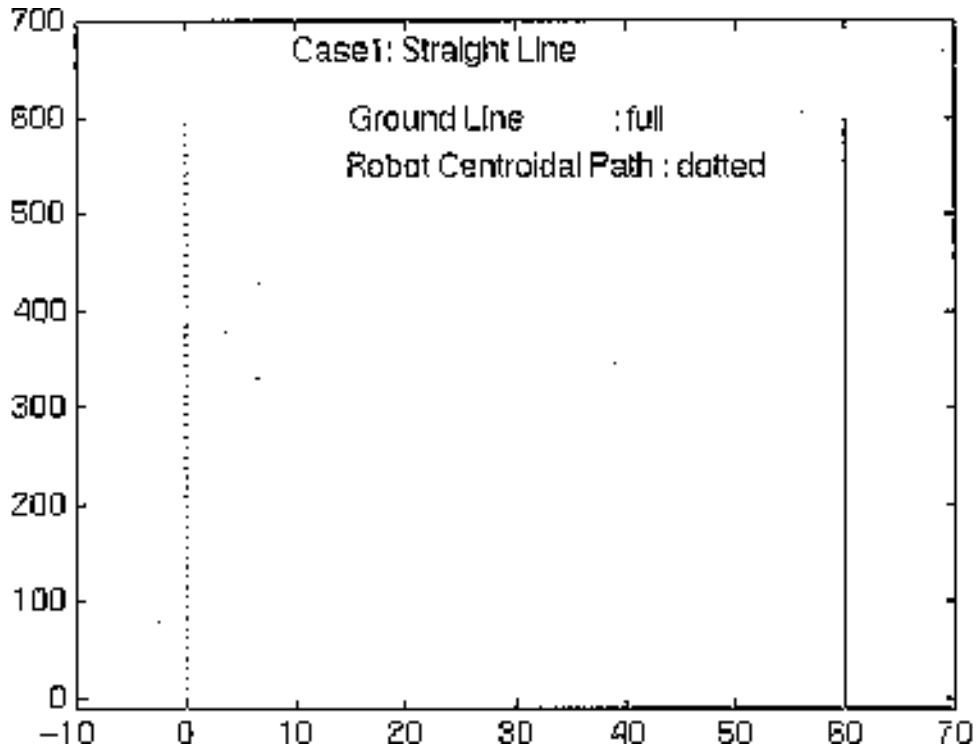


Figure 12. Case 1

The objective of this case was to test if the controller handles a simple case as straight line following. The actual line is represented by a solid straight line and the actual path of the robot (the centroidal path) is represented by the dotted line in all the cases. The resultant output of this simulation is presented in Figure 12. The output indicates a successful line following.

Case 2: Curved Path

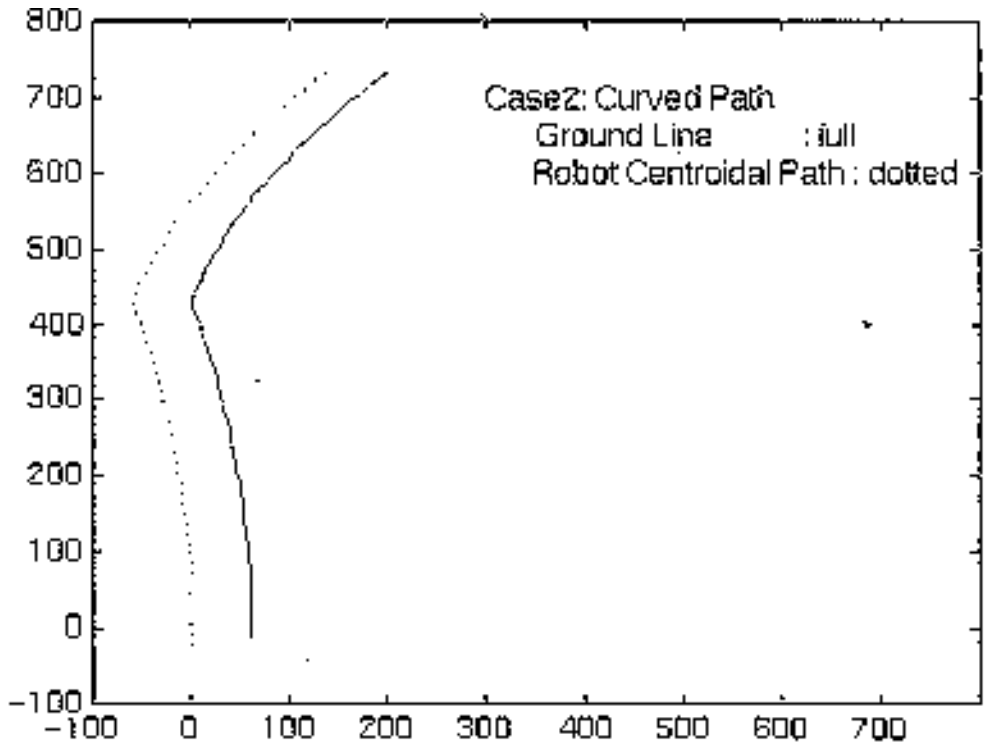


Figure 13. Case 2

In Case 2, the robot was made to follow a curved line. In this case the input data was free of any noise in form of obstacles and loss of vision. The result of this simulation as seen in Figure 13, suggests that the fuzzy inference system was able to direct the vehicle along curved lines.

Case 3: Sensitivity to distance variation

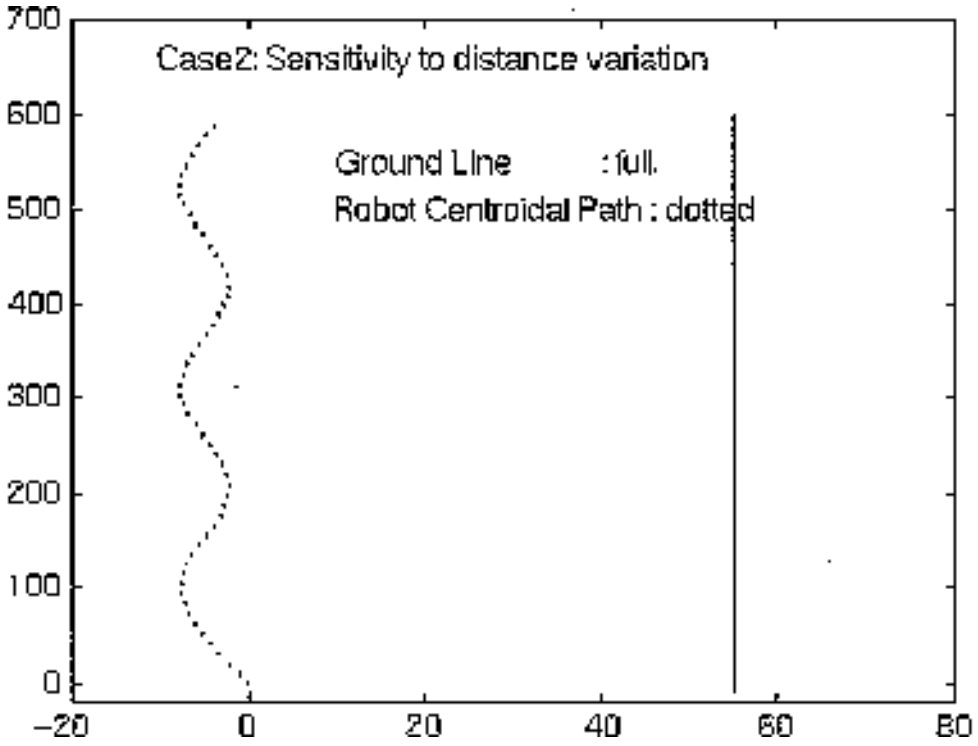


Figure 14. Case 3

In Case 3, noise was introduced into the controllers input data in form of loss of vision input. The fuzzy inference system was found to be very sensitive to the noise. The resultant output was a wavy pattern suggesting overshoot and lack to proper damping. This pattern was observed only in this case. A solution for this problem is to adjust the rules and the membership functions to take into account this unstable condition. But, this modification has its effect on the remaining cases. This presents a limitation of the fuzzy inference system.

Case 4: Sensitivity to angular fluctuations

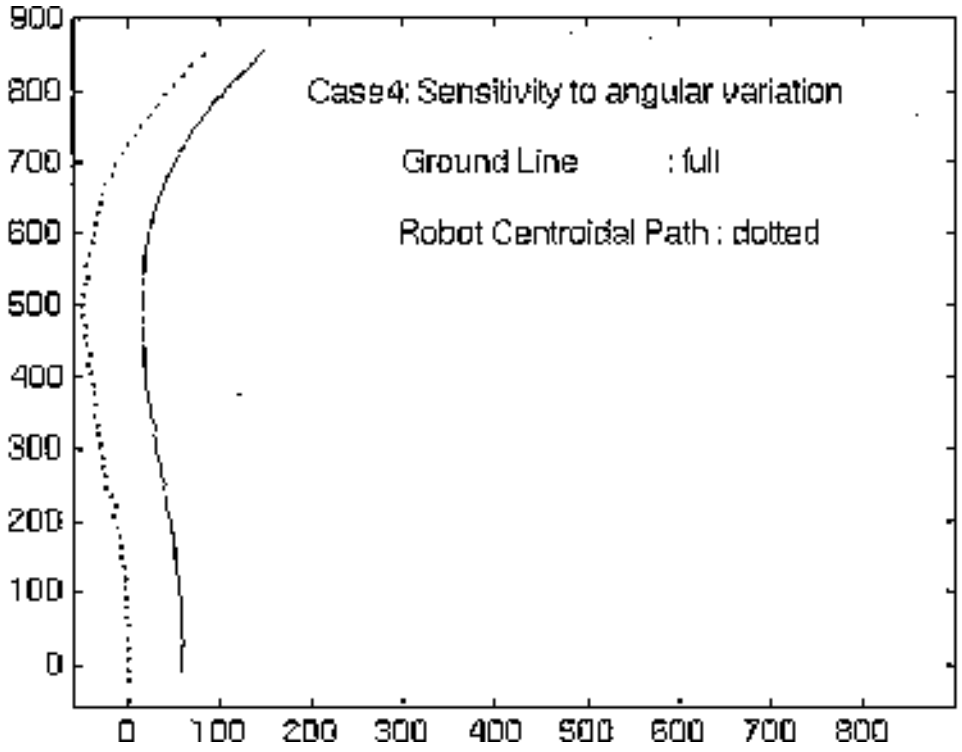


Figure 15. Case 4

In Case 4, the robot followed a curved line. Noise in form of loss of input vision data and obstacle was used during data collection stage. The simulation presents a successful line following. The inference engine worked successfully for this case.

Case 5: Extreme environment

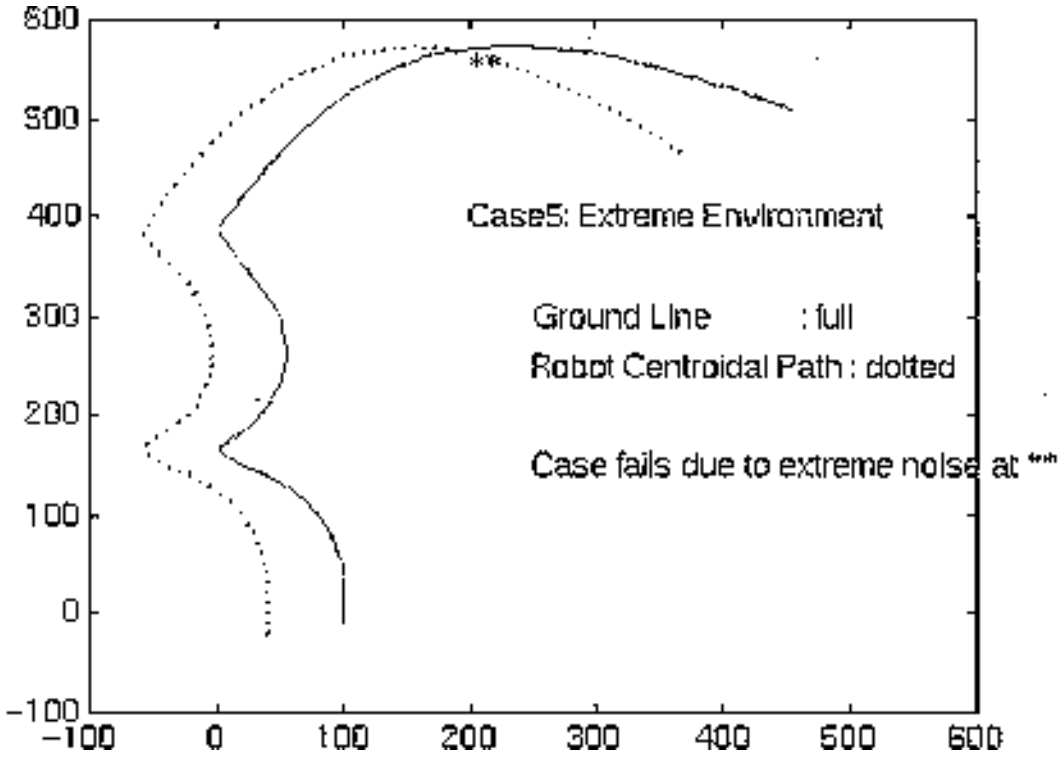


Figure 16. Case 5

In Case 5, an extreme environment with excessive noise in form of adverse vision input data and obstacles in sensitive positions (on curves as pointed out in the simulation) was introduced. To make this case more difficult steep curves were used. The simulation presents a failure at the point when the robot path crosses the line it is supposed to follow. This presents a limitation of the fuzzy inference engine. This limitation arises due to the limitation on the number of rules that can be implemented in this present controller. The FLC fails due to the excess adverse parameters introduced in this case. A solution to this problem is to use a neural network to identify such

extreme conditions and use dedicated fuzzy inference engines for each case identified by the neural network. This solution has been discussed on in Chapter 7.

Chapter 7. Conclusion and Recommendations

Conclusion

The design and implementation of a modular fuzzy logic based controller for an autonomous mobile robot for line following along an obstacle course has been presented. The control algorithm for this application is based on vision navigation. The development of the FLC controller was done after a detail study of the autonomous guided vehicle and its environment. A rule base was generated using expert system knowledge. Fuzzy membership functions and fuzzy sets were developed. The FLC model was first tested on the MATLAB fuzzy logic toolkit with some special cases. The FLC was then implemented in C programming language and a number of tests were run to analyze the stability and response of the system under fuzzy control in a real life scenario. Tuning of the system in form of adjusting the membership functions and the rules was done to improve the stability of the FLC.

The fuzzy logic control is a very flexible and robust soft computing tool for control. The number of variants involved in the current application present a challenge for any type of control system. A fuzzy logic control was selected, as a soft computing solution for this problem keeping in mind its robustness and flexibility. The performance of the robot was studied with simulations for five different cases selected for the study. The FLC shows good stability and response for three of the cases. The problem at hand seems to be a complex problem for just one inference engine to handle. This limitation arises due to the limit on the number of rules and membership functions

that can be used in a single inference engine. A better system performance can be obtained if a neuro-fuzzy approach is used. The environment in which the robot runs should be divided into a number of specific classes according to input data. The control system model will contain a neural net to identify and classify input data, which will finally fire the right inference engine for the input data class. From the results obtained in the MATLAB simulation and the preliminary testing of the model on the robot, it can be concluded that the model presented, can be reliably and successfully implemented permanently on the robot.

Fuzzy logic has been proven to be a excellent solution to control problems where the number of rules for a system are finite and which can be easily established. In this application an infinite number of rules can be established. The fuzzy control in a way acts as a learning system control, as it has the ability to learn from situations where it fails. This learning is possible by increasing the number of rules in the system. In this way the system can keep on learning until it becomes a perfect system. No system is perfect in the onset; as such the necessary refinements and modifications will be made to make the UC Bearcat a competitive mobile robot.

Future Recommendations

The FLC has proven to be a robust controller for this system. FLC can be successfully implemented in systems which need robust performance. The FLC is stable under all the cases discussed in Chapter 6. Tuning of the FLC is a challenging task and needs an expert knowledge. The total error of the control loop can be reduced according to the needs of a system by tuning the FLC. The tuning process is the modification of fuzzy sets and fuzzy rule base. The shape of the membership function has a negligible effect on the final stability of the system. Future research should be directed to the development of an easier tuning method for the fuzzy system. Also, alpha cut and clustering algorithms could be used for better performance of the fuzzy engine.

It was observed during this study that, this particular control problem had some special cases which could be eventually handled more effectively by separate fuzzy inference engines. A neuro-fuzzy approach should be tried with this problem. A neural network should be used to identify special cases which arise during the motion of the robot on the path. Then a separate FLC should be used to control each case independently. This will improve the performance of the system with regards to the stability aspect.

Future considerations might also include experimenting with different fuzzy implications and operators for this particular application. This fuzzy approach can also

be used in other aspects of the system such as safety, error anticipation and handling and fault reporting.

References

[1] P. F. Muir and C. P. Neuman, 'Kinematic Modeling of Wheeled Mobile Robots,' **Journal of Robotic Systems**, 4(2), 1987, pp. 281-340

[2] E. L. Hall and B. C. Hall, **Robotics: A User-Friendly Introduction**, Holt, Rinehart, and Winston, New York, NY, 1985, pp. 23.

[3] Z. L. Cao, S. J. Oh, and E. L. Hall, "Dynamic omni-directional vision for mobile robots," **Journal of Robotic Systems**, 3(1), 1986, pp. 5-17.

[4] Z. L. Cao, Y. Y. Huang, and E. L. Hall, "Region Filling Operations with Random Obstacle Avoidance for Mobile Robots," **Journal of Robotics Systems**, 5(2), 1988, pp. 87-102.

[5] S. J. Oh and E. L. Hall, "Calibration of an omni-directional vision navigation system using an industrial robot," **Optical Engineering**, Sept. 1989, Vol. 28, No. 9, pp. 955-962.

[6] R. M. H. Cheng and R. Rajagopalan, "Kinematics of Automated Guided Vehicles with an Inclined Steering Column and an Offset Distance: Criteria for Existence of Inverse Kinematic Solution," **Journal of Robotics Systems**, 9(8), Dec. , 1992, 1059-1081.

[7] M. P. Ghayalod, E. L. Hall, F. W. Reckelhoff, B. O. Mathews and M. A. Ruthemeyer, "Line Following Using Omni-directional Vision," Proc. of SPIE

Intelligent Robots and Computer Vision Conf., SPIE Vol. 2056, Boston, MA, 1993, page 101.

[8] Kazuo Tanaka, “Design of Model-based Fuzzy Controller Using Lyapunov’s Stability Approach and Its Application to Trajectory Stabilization of a Model Car,” **Theoretical Aspects of Fuzzy Control**, John Wiley & sons, 1995. 2nd IEEE conference on fuzzy system, San Francisco, CA, 1993, Inc, pp.31-50.

[9] C. V. Altrock et al., “Advanced fuzzy logic control technologies in automotive applications”, **Proceedings of 1st IEEE international Conference on Fuzzy Systems**, 1992, pp. 835-842.

[10] L. A. Zadeh, "Outline of a new Approach to the Analysis of Complex Systems and Decision Process", 1973 **IEEE Transacions on Systems, Man and Cybernetics**, v3, pp 28-44.

[11] E. H. Mamdani, "Fuzzy Sets for Man - Machine Interaction ", 1977, **IEEE Transactions on Computer**, v26, pp. 1182-1191.

[12] J. F. Baldwin and N. C. F. Guild, "Feasible Algorithms For Approximate Reasoning Using Fuzzy Logic", 1980, **Fuzzy Sets and Systems**, v3, pp. 225-251..

[13] J. F. Baldwin and B. W. Pilsworth, "Axiomatic Approach For Approximate Reasoning with Fuzzy Logic", 1980, **Fuzzy Sets and Systems**, v3, pp. 193-219

[14] J. B. Kiszka, M. M. Gupta and G. M. Trojan, "Multi Variable Fuzzy Controller under Goedel's Implication", **Fuzzy Sets and Systems**, v34, pp. 301-321.

[15] Z. Cao and A. Kandel, "Applicability of some Fuzzy Implication Operators", 1989, **Fuzzy Sets and Systems**, v31, pp. 151-186.

[16] A. Bardossy, L. Duckstein, "Fuzzy Rule based Modeling with Application to Geophysical, Biological and Engineering Systems", 1995, CRC Press, New York, pp.62-68.

[17] H. Hellendoorn, R. Palm, "Fuzzy Systems Technologies at Siemens R & D", 1994, **Fuzzy Sets and Fuzzy Systems**, Vol. 63, pp. 245-269.

[18] M. Jamshidi, N. Vadiee, T. Ross, "Fuzzy Logic Control", 1993, PTR Prentice Hall, Englewood Cliffs, New Jersey 07632, pp 89-101.

[19] P. King, E. Mamdani, "The application of Fuzzy Control Systems to Industrial Process", 1977, **Automatica**, Vol. 3, pp. 235-242.

[20] R. Stenz, U. Kuhn, "Automation of a batch distillation column using Fuzzy and Conventional Control", 1995 **IEEE Transactions on Systems Technology**, Vol. 3, pp. 171-176.

[21] T. Takahashi, M. Kitou, M. Asai, M. Kido, T. Chiba, J. Kawakami, Y. Matsui, "A new voltage equipment using Fuzzy Inference", 1994, **Electrical Engineering in Japan**, Vol. 114, pp. 18-32.

[22] C. von Altrock, Fuzzy Logic and NeuroFuzzy Applications Explained Prentice Hall PTR, Englewood Cliffs, NJ, 07632, page 78.

[23] J. Yen, R. Langari, L. Zadeh, "Industrial applications of Fuzzy Logic and Intelligent Systems", 1995, **IEEE Press**, IEEE Inc, New York, pp 08-09.

[24] T. Yamakawa, "Stabilization of an inverted pendulum by a high-speed Fuzzy Logic Controller Hardware System", **Fuzzy Sets and Systems**, Vol. 32, pp. 161-180.

[25] Charles P. Coleman and Datta Godbole, University of California at Berkeley “A Comparison of Robustness: Fuzzy Logic, PID, & Sliding Mode Control”, 1996, pp.06-08

[26] Z. Yuzhou, R. McLauchlan, R. Challoo and S. Omar "Fuzzy Logic Control of a four-link Robotic Manipulator in a Vertical Plane" Proceedings of the Artificial Neural Networks in Engineering (**ANNIE '96**) Conference, held November 10-13, 1996, St. Louis US, pp 198-200.

[27] J.-S. Roger Jang and Ned Gulley, **Fuzzy Logic Toolbox For Use with MATLAB**, The MathWorks Inc. 1995, pp 90-92.

[28] Bart Kosko “Neural Networks and Fuzzy Systems- A Dynamical Systems Approach To Machine Intelligence”, Prentice Hall, Englewood Cliffs, NJ 07632, pp 45-67.

[29] N. Gulley "How To Design Fuzzy Logic Controllers" from Machine Design, November 1992, page 26.

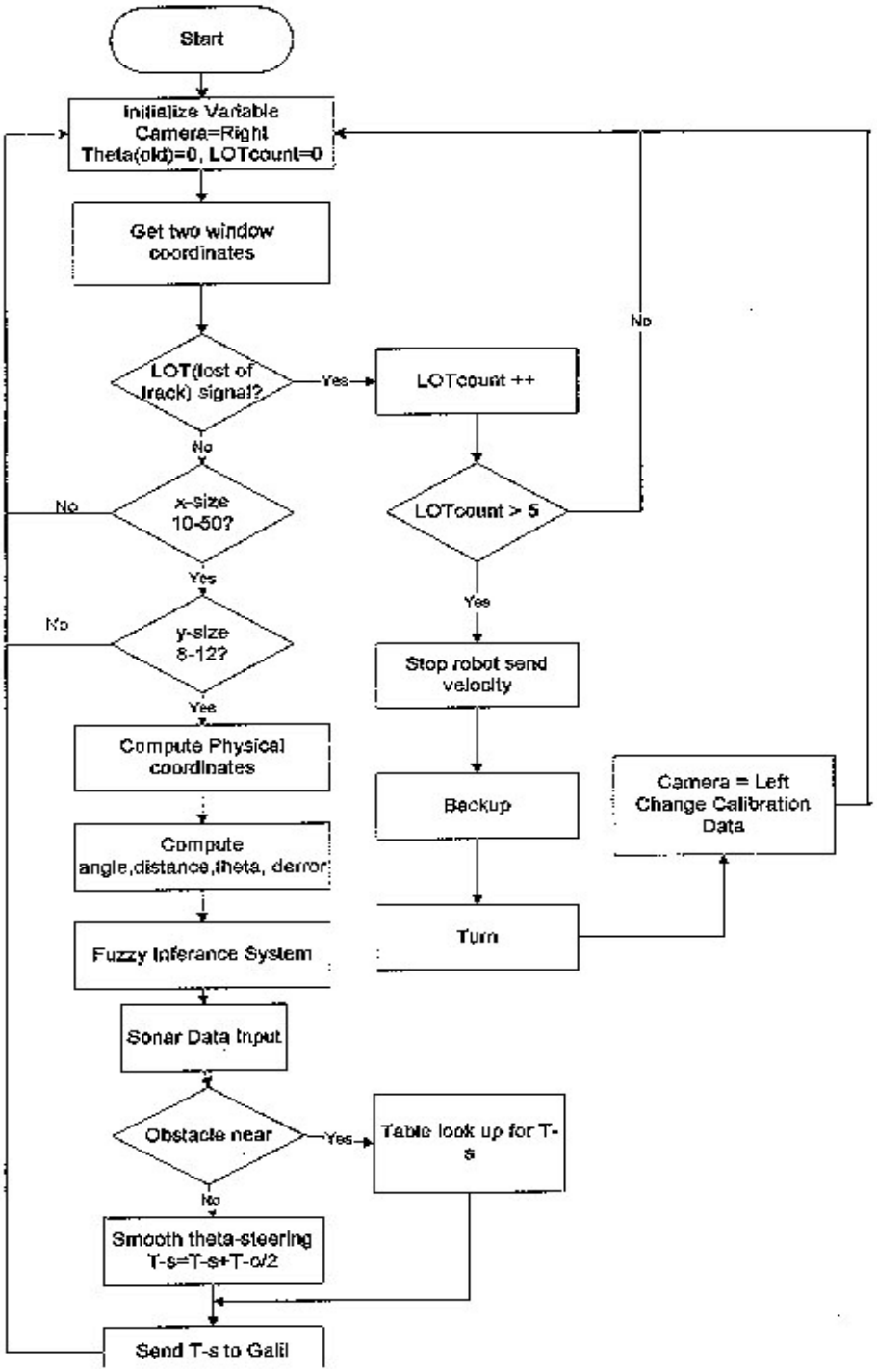
[30] Mark Kantrowitz, Erik Horstkotte, and Cliff Joslyn “The Internet fuzzy-logic FAQ ("frequently-asked questions") list” postings for comp.ai.fuzzy URL:ftp.cs.cmu.edu:/user/ai/pubs/faqs/fuzzy/fuzzy_1.faq, 1996

[31] Kevin Self "Designing With Fuzzy Logic" from **IEEE SPECTRUM**, November 1990, Volume 105 pp 42-44.

[32] T. Takagi, M. Sugeno, "Fuzzy Identification of Systems and it's Application to Modeling and Control", 1985, **IEEE Transactions on Systems**, Man, and Cybernetics, V.SMC-15, pp. 116-132.

Appendix

Flow Chart for the control algorithm



Fuzzy logic controller: C Program.

Header Files:
Point.h

```
/* This is a point definition */

typedef struct {

double x;

double y;

} Point;


Point * IniPoint( double dx, double dy);
```

Vector.h

```
/* Definitions for an object vector*/

#include <math.h>

typedef struct {

double dx;

double dy;

} Vector;


double magnitude(Vector * X);

double VectorDot( Vector * A , Vector * B);

double VectorCrossMag( Vector * A, Vector * B);

Vector * IniVector( Point * A, Point * B);
```

```
Vector * UnitVector( Vector * N );

Vector * UnitVectorXaxis();
```

Vectors.h

```
#include "point.h"

#include "vector.h"

#include "fcontrol.h"
```

Fcontrol.h

```
/* header file for the fcontrol function..Fuzzy Controller.*/

#include "fuzzin.h"

double Fcontrol( double x, double phi);
```

Fuzzin.h

```
#include <stdio.h>

double fuzzin ( double x, double left, double mid, double right );

double SumMatrix ( int rules[][5], double state[][5] );
```

Main control code

```
/* Using the point and the vector header files.*/

#include <stdio.h>

#include <stddef.h>

#include <alloc.h>

#include <string.h>

#include <math.h>

#include "vectors.h"


void main(){

        FILE *data;

        int count;

        float distance;

        double dx1, dy1, dx2, dy2, dx3, dy3, dx4, dy4, temp, temp1;

        double phi, phi1, addangle, angle;

        double test1, test2; /* debug values*/

        double dist1, dist2, offset;

        char buf[40];


        Point * R1, * R2, * G1, * G2, * G1_old, * G2_old;

        Vector * N1, * N2, * unitx;


/* Initialize variables */
```

```
        count = 0; /* counter for number of cycles */
        phi1 = 0;
        R1 = (Point *) malloc(sizeof(Point));  /* Robot centroid*/
        R2 = (Point *) malloc(sizeof(Point));  /* Robot Steering position */
        G1 = (Point *) malloc(sizeof(Point));  /* Ground 1 */
        G2 = (Point *) malloc(sizeof(Point));  /* Ground 2 */
        G1_old = (Point *) malloc(sizeof(Point));  /* Old ground points */
        G2_old = (Point *) malloc(sizeof(Point));
        N1 = (Vector *) malloc(sizeof(Vector));
        N2 = (Vector *) malloc(sizeof(Vector));
        unitx = (Vector *) malloc(sizeof(Vector));

        if( (data=fopen("data.dat","r")) == NULL )
        puts("Can not find the file data.dat");

        while( fgets(buf, 30, data) > 0 ){
/* To start with I have 8 point input Otherwise just 4 points */
                if(count == 0){
                sscanf( buf,"%lf %lf %lf %lf %lf %lf %lf %lf
",&dx1,&dy1,&dx2,&dy2,&dx3,&dy3,&dx4,&dy4);

                R1 = IniPoint( dx1, dy1);
                R2 = IniPoint( dx2, dy2);
```

```
                }
                else
                sscanf( buf,"%lf %lf %lf %lf ",&dx3,&dy3,&dx4,&dy4);

                G1 = IniPoint( dx3, dy3);
                G2 = IniPoint( dx4, dy4);
                N1 = IniVector( R1, R2);
                N2 = IniVector( G1, G2);
                temp = magnitude( N1 ) * magnitude( N2 );
                temp1 = VectorCrossMag( N1, N2);
                temp = temp1 / temp ;

                phi = asin(temp);

/* Compute the addition angle */
                unitx = UnitVectorXaxis();
                temp = magnitude( N1 );
                addangle = VectorCrossMag( unitx, N1);
                addangle = addangle / temp;
                addangle = asin(addangle);
                addangle = addangle * 180 / 3.14;
                printf("Input R1:%lf,%lf and R2:%lf,%lf",R1->x,R1->y,R2->x,R2-
>y);
```

```
                printf("Input G1:%lf,%lf and G2:%lf,%lf \n",G1->x,G1->y,G2-
>x,G2->y);
                printf("Angle computed is: %lf\n",phi*180/3.14);
//              printf("AddAngle computed is: %lf\n",addangle);

/* Calculate distance error*/
/* Shift to local co-ordinate system and calculate the x distance with respect to
the last point */

/* Local Co-ordinates */
                if(count != 0){
                dist1 = sqrt( ((G2_old->x - R2->x) * (G2_old->x - R2->x)) +
((G2_old->y - R2->y) * (G2_old->y - R2->y)) );
//              dist2 = ( G2_old->x - R1->x );

/* Distance is the mean x position */

//              distance = (float) ((dist1 + dist2) / 2 );
                distance = dist1;

//              printf("Distance computed is: %lf ",distance);
                }
/* Error distance */
```

```
                if(count == 0)
                distance = 0;
                else
                distance = distance - 60;
                printf("Error distance is: %lf and angle %lf ",distance, -
phi*180/3.14);
/* Call the fuzzy inferance engine */

                if(distance != 0){
                phi1 = Fcontrol(distance, -phi*180/3.14);
                phi1 = -phi1;
                printf("Returned angle: %lf\n",phi1);
                }
                else
                phi1 = phi*180/3.14;

/* Steer Car */


/* Calculate the new Robot co-ordinates */
/* Calculate offset ie. the distance between G1 and G2 ie, magnitude of N2 */

                offset = magnitude(N2);
```

```
//              printf("Offset of the new co-ordinate is: %lf\n",offset);
                angle = phi1 + addangle;
                R1->x = R2->x;
                R1->y = R2->y;
                R2->x = R1->x + ( offset * cos(angle * 3.14 / 180) );
                R2->y = R1->y + ( offset * sin(angle * 3.14 / 180) );

                printf("New R1:%lf,%lf and R2:%lf,%lf \n",R1->x,R1->y,R2->x,R2-
>y);

/* Save the current ground co-ordinates as old co-ordinates. */

                G1_old->x = G1->x;
                G1_old->y = G1->y;
                G2_old->x = G2->x;
                G2_old->y = G2->y;

//              printf("Old Ground data G1:%lf,%lf and G2:%lf,%lf \n",G1->x,G1-
>y,G2->x,G2->y);

/* Return in the while loop. */
```

```
                count++;

                buf[0] = '\0';    /* buffer is reinitialized for the next cycle.*/

/* Debug output*/

                printf("End----------------------------------:\n");



                } // End of the while loop.

        printf("Counts completed %d\n",count);

        fclose(data);

} // End of this main code.
```

Fuzzy Controller

```
#include <stdio.h>
#include <math.h>
#include <string.h>
#include "fcontrol.h"

double Fcontrol(double x1, double phi1) {
```

```
/* Fuzzy rules----------------------------------------------------------------*/

        const char FAM[7][5][2]={ "BN", "BN", "MN", "SN", "SP",
                                  "BN", "MN", "MN", "SN", "SP",
                                  "MN", "MN", "SN", "SP", "SP",
                                  "SN", "SN", "ZE", "SP", "SP",
                                  "SN", "SN", "SP", "MP", "MP",
                                  "SN", "SP", "MP", "MP", "BP",
                                  "SN", "SP", "MP", "BP", "BP"};

/* Binary integer matrices....*/

        int iBN[7][5], iMN[7][5], iSN[7][5], iZE[7][5], iSP[7][5],
        iMP[7][5], iBP[7][5];

/* Temporary string variables for comparison purpose.*/

        char *cBN = "BN";
        char *cMN = "MN";
        char *cSN = "SN";
        char *cZE = "ZE";
        char *cBP = "BP";
```

```
        char *cMP = "MP";
        char *cSP = "SP";
/* ufsets are the output fuzzy sets----------------------------*/

        const double ufset[]={ -20.0, -12.0, -5.0, 0.0, 5.0, 12.0, 20.0 };
/* ufareas are the area of the corresponding fuzzy membership functions---*/

        const double ufarea[]={ 5.0, 8.0, 5.0, 4.0, 5.0, 8.0, 5.0 };
        int i,j,k;
        char crulestr[3];
/* Variables for internal use.*/

        double  phif[7], xf[5], fzstate[7][5],  uf[7], temp, u;
/* Initialize matrices */
        for(i=0; i<3; i++)
        crulestr[i] = '\0';

        for(i=0; i<7; i++){
                for(j=0; j<5; j++){
                iBN[i][j] = iMN[i][j] = iSN[i][j] = 0;
                iZE[i][j] = 0;
                iBP[i][j] = iMP[i][j] = iSP[i][j] = 0;
                }
```

```
        }
/* Construct the individual FAM matrix for each output fuzzy set------*/


        for ( i=0; i<7 ;i++){
                for ( j=0; j<5; j++){
                        for ( k=0; k<2; k++){
                        crulestr[k] = FAM[i][j][k];
                        }
                        if ( strcmp(crulestr, cBN) == 0 )
                        iBN[i][j] = 1;
                        if ( strcmp(crulestr, cMN) == 0 )
                        iMN[i][j] = 1;
                        if ( strcmp(crulestr, cSN) == 0 )
                        iSN[i][j] = 1;
                        if ( strcmp(crulestr, cZE) == 0 )
                        iZE[i][j] = 1;
                        if ( strcmp(crulestr, cBP) == 0 )
                        iBP[i][j] = 1;
                        if ( strcmp(crulestr, cMP) == 0 )
                        iMP[i][j] = 1;
                        if ( strcmp(crulestr, cSP) == 0 )
                        iSP[i][j] = 1;
//                      printf("Rulestr is %s\n", crulestr);
```

```
            }
        }
/* fuzzify input data by fuzzy membership function and store in local variables*/

            xf[0]=fuzzin (x1, -36, -36, -20);          /*d-2*/
            xf[1]=fuzzin (x1, -36, -18, 0);           /*d-1*/
            xf[2]=fuzzin (x1, -5, 0, 5);              /*d*/
            xf[3]=fuzzin (x1, 0, 18, 36);              /*d1*/
            xf[4]=fuzzin (x1, 20, 36, 36);             /*d2*/

            phif[0]=fuzzin (phi1,-30, -20, -15);             /*a-3*/
            phif[1]=fuzzin (phi1, -20, -15, -10);       /*a-2*/
            phif[2]=fuzzin (phi1, -15, -10, -5);        /*a-1*/
            phif[3]=fuzzin (phi1, -8, 0, 8);           /*a*/
            phif[4]=fuzzin (phi1, 5, 10, 15);           /*a1*/
            phif[5]=fuzzin (phi1, 10, 15, 20);          /*a2*/
            phif[6]=fuzzin (phi1, 15, 20, 30);          /*a3*/
/* construct fuzzy state vectors-------------------------------------------------*/
/* combine two input fuzzy membership together to get one output fuzzy
membership */
/* for each fuzzy rule. Max encoding.                                        */

            for (i=0; i <= 6; ++i)
```

```
            for (j=0; j <= 4; ++j)
              fzstate[i][j] = phif[i] * xf[j];          /*algebraic product*/
/* construct fuzzy sets-------------------------------------------------------*/
/* get membership sum for the same output fuzzy set */

        uf[0] = SumMatrix (iBN, fzstate);               /*uNB*/
        uf[1] = SumMatrix (iMN, fzstate);               /*uNM*/
        uf[2] = SumMatrix (iSN, fzstate);               /*uNS*/
        uf[3] = SumMatrix (iZE, fzstate);       /*uZE*/
        uf[4] = SumMatrix (iSP, fzstate);               /*uPS*/
        uf[5] = SumMatrix (iMP, fzstate);               /*uPM*/
        uf[6] = SumMatrix (iBP, fzstate);               /*uPB*/
/*centroid defuzzification---------------------------------------------------*/

        temp = 0.0;
        for (i=0; i<=6; ++i)
            temp = temp + uf[i] * ufarea[i];       /* Product of the area and uf[]*/

        if ((temp > -1e-8) && (temp < 1e-8))
            u = 0.0;                            /*in case divided by zero*/
        else
        {
            u = 0.0;
```

```
                for (j=0; j<=6; ++j)
                 u = u + ufset[j] * uf[j] * ufarea[j];
                 u = u / temp;
                }
          if (u > 30.0)
                u = 30.0;
          else if (u < -30.0)
                u = -30;
//        printf("Fuzzy output value will be: %f\n", u);
          return u;
} //End of the procedure.
```

Fuzzification algorithm used by the fuzzy controller

```
#include <stdio.h>
#include <math.h>
#include "fuzzin.h"
double fuzzin ( double x, double left, double mid, double right ){
double memship;
    if ( left == mid ){
        if ( x <= left )
          memship = 1;
        else if ( x >= right )
          memship = 0;
        else
          memship = (x - right)/(mid - right);
        }

    else if ( mid == right )
        {
        if ( x >= right )
          memship = 1;
        else if ( x <= left )
          memship = 0;
        else
          memship = (x - left) / (mid-left);
```

```
        }


    else
        {
        if ( x <= left || x >= right )
           memship = 0;
        else if ( x <= mid )
           memship = (x - left) / (mid - left);
        else
           memship = (x - right) / (mid-right);
        }

     return (memship);
  }


double SumMatrix ( int rules[][5], double state[][5] )
   { int i, j;
     double result = 0.0;
         for (i=0; i<=6; ++i)
           for (j=0; j<=4; ++j)
                result = result + rules[i][j] * state[i][j];
         return (result);
   }
```

Vector and Point Operations

```
#include "Point.h"
#include <alloc.h>

/* Operations with the Point struct*/
Point * IniPoint( double dx, double dy){
Point * ret;
ret = (Point *) malloc(sizeof(Point));
ret->x = dx;
ret->y = dy;
return ret;
}

#include <math.h>
#include "Point.h"
#include "Vector.h"
#include <alloc.h>

/* Operations on a vector */
double magnitude(Vector * X){
double dmag=0;
dmag=sqrt((X->dx * X->dx) + (X->dy * X->dy));
```

```
return dmag;
}//End of the magnitude function.

/* dot product */
double VectorDot( Vector * A , Vector * B){
        double dDot_Product=0.0;

        dDot_Product = A->dx * B->dx + A->dy * B->dy;
        return dDot_Product;
}

/* cross product */
double VectorCrossMag( Vector * A, Vector * B){
        double ret;
        ret = (A->dx * B->dy) - (B->dx * A->dy);
        return ret;
}

Vector * IniVector( Point * A, Point * B){

        Vector * ret;

        ret = (Vector *) malloc(sizeof(Vector));
```

```
	ret->dx = (B->x - A->x);
	ret->dy = (B->y - A->y);

	return ret;
}
Vector * UnitVectorXaxis(){

	Vector * ret;
	ret = (Vector *) malloc(sizeof(Vector));
	ret->dx = 1;
	ret->dy = 0;

	return ret;
}

Vector * UnitVector( Vector * N ) {

	Vector * ret;
	ret = (Vector *) malloc(sizeof(Vector));
	ret->dx = ( N->dx / ( magnitude( N )));
	ret->dy = ( N->dy / ( magnitude( N )));
	return ret;
```

```
}
```

Steering angle output table for different distance and angle inputs.

Angle: \ **Distance:**	*-36*	*-30*	*-24*	*-8*	*0*	*8*	*24*	*30*	*36*
-20	20.00	20.00	12.00	5.00	5.00	5.00	5.00	5.00	5.00
-15	20.00	16.32	12.00	5.00	5.00	5.00	5.00	1.52	1.52
-10	20.00	13.52	12.00	5.00	5.00	5.00	5.00	2.27	2.27
-5	20.00	12.00	12.00	5.00	5.00	5.00	5.00	5.00	5.00
0	12.00	12.00	5.00	0.00	0.00	0.00	5.00	12.00	12.00
5	5.00	5.00	5.00	5.00	5.00	5.00	12.00	12.00	20.00
10	2.27	2.27	5.00	5.00	5.00	5.00	12.00	13.52	20.00
15	1.52	1.52	5.00	5.00	5.00	5.00	12.00	16.32	20.00
20	5.00	5.00	5.00	5.00	5.00	5.00	12.00	20.00	20.00

M file for this simulation:

```
% Create input set for the first level

input = [1.5 10; -2.0 13; -2.8 5; 1.8 -20;
        -2.2 -15; 0.8 15; 0 -12; 3 19;
         1.2 -8; 2 0];
% Evaluate the output of the first-level

  fuzzy-sys-1 = readfis('file1');
  Steering-ang = evalfis(input, fuzzy-sys-1);

% Input the steering angle to the second fuzzy level
% to compute the speed.

  fuzzy-sys-2 = readfis('file2);
  speed = evalfis(steering-ang, fuzzy-sys-2);
```

Fuzzy Associated Memories

Multivalued fuzzy sets can be represented as points in a unit hypercube.

$$I^n = [0,1]^n$$

These cubes can be used as a measure of how much one fuzzy set is a subset of any other fuzzy set. These fuzzy cubes can be mapped. This level of abstraction provides a surprising and fruitful alternative to the proportional and predicate calculus reasoning techniques used in the artificial-intelligence expert systems.

The simplest FAM encodes the FAM rule or association (A_i,B_i), which associates a p - dimensional fuzzy set B_i with the n - dimensional fuzzy set A_i. These minimal FAMs essentially map one ball in I^n with one ball in I^p.

A general FAM system:

$$F: I^n \rightarrow I^p$$

A FAM system encodes and process in parallel a FAM bank of m rules $(A_1,B_1)(A_2,B_2)....(A_m,B_m)$. Each input A to the FAM system activates each stored FAM rule to different degree.

FAM rules can be represent compound linguistic conditionals. For example $(A^1,A^2;B)$ means in linguistic terms as

"IF X^1 is A^1 and X^2 is A^2 THEN Y is B".

Matlab Simulation data

Case 1:

0 -10 0 0 60 -10 60 0		60.000000	10.00000	60.000000	20.00000	60.000000	30.00000
60.000000	40.00000	60.000000	50.00000	60.000000	60.00000	60.000000	70.00000
60.000000	80.00000	60.000000	90.00000	60.000000	100.0000	60.000000	110.0000
60.000000	120.0000	60.000000	130.0000	60.000000	140.0000	60.000000	150.0000
60.000000	160.0000	60.000000	170.0000	60.000000	180.0000	60.000000	190.0000
60.000000	200.0000	60.000000	210.0000	60.000000	220.0000	60.000000	230.0000
60.000000	240.0000	60.000000	250.0000	60.000000	260.0000	60.000000	270.0000
60.000000	280.0000	60.000000	290.0000	60.000000	300.0000	60.000000	310.0000
60.000000	320.0000	60.000000	330.0000	60.000000	340.0000	60.000000	350.0000
60.000000	360.0000	60.000000	370.0000	60.000000	380.0000	60.000000	390.0000
60.000000	400.0000	60.000000	410.0000	60.000000	420.0000	60.000000	430.0000
60.000000	440.0000	60.000000	450.0000	60.000000	460.0000	60.000000	470.0000
60.000000	480.0000	60.000000	490.0000	60.000000	500.0000	60.000000	510.0000
60.000000	520.0000	60.000000	530.0000	60.000000	540.0000	60.000000	550.0000
60.000000	560.0000	60.000000	570.0000	60.000000	580.0000	60.000000	590.0000
60.000000	600.0000						

Case 2:

0 -10 0 0 60 -10 60 0		60.381130	9.996066	60.625760	19.99071	60.739120	29.98970
60.726170	39.98904	60.592040	49.98689	60.341870	59.98335	59.980880	69.97624
59.514300	79.96444	58.947310	89.94835	58.285250	99.92539	57.533250	109.8968
56.696580	119.8614	55.780470	129.8188	54.790020	139.7696	53.730490	149.7129
52.606990	159.6490	51.424500	169.5793	50.188370	179.5017	48.903250	189.4183
47.571000	199.3280	46.191020	209.2320	44.763050	219.1286	43.286530	229.0183
41.761120	238.9005	40.186320	248.7752	38.561890	258.6412	36.887130	268.5002

0.0 0.0 0.0 0.0
35.16222 278.3485 33.38622 288.1893 31.55919 298.0200 29.68068 307.8409
0.0 0.0 0.0 0.0
27.75009 317.6532 25.76717 327.4529 23.72544 337.2433 21.61828 347.0183
0.0 0.0 0.0 0.0
19.43789 356.7775 17.17668 366.5190 14.82782 376.2377 12.38260 385.9357
0.0 0.0 0.0 0.0
9.835211 395.6036 7.176231 405.2454 4.400334 414.8501 1.497960 424.4211
0.0 0.0 0.0 0.0
1.536131 433.9476 4.711085 443.4309 8.032356 452.8615 11.50820 462.2379
0.0 0.0 0.0 0.0
15.14422 471.5517 18.94332 480.8001 22.90264 489.9831 27.01848 499.1001
0.0 0.0 0.0 0.0
31.28540 508.1471 35.69552 517.1157 40.25059 526.0171 44.94694 534.8508
0.0 0.0 0.0 0.0
49.77941 543.6137 54.73584 552.2910 59.82192 560.8993 65.03402 569.4383
0.0 0.0 0.0 0.0
70.36829 577.9072 75.81108 586.2902 81.36849 594.6026 87.03690 602.8440
0.0 0.0 0.0 0.0
92.81268 611.0137 98.68860 619.1066 104.6641 627.1263 110.7359 635.0731
0.0 0.0 0.0 0.0
116.9004 642.9465 123.1601 650.7533 129.5072 658.4877 135.9364 666.1471
0.0 0.0 0.0 0.0
142.4439 673.7308 149.0425 681.2567 155.7214 688.7155 162.4683 696.0965
0.0 0.0 0.0 0.0
169.2795 703.3994 176.1772 710.6506 183.1541 717.8442 190.1849 724.9570
0.0 0.0 0.0 0.0
197.2659 731.9885
0.0

Case 3:

0 -10 0 0 55 -10 55 0 55.00000 10.00000 55.00000 20.00000 55.00000 30.00000
0.0 0.0 0.0 0.0
55.00000 40.00000 55.00000 50.00000 55.00000 60.00000 55.00000 70.00000
0.0 0.0 0.0 0.0
55.00000 80.00000 55.00000 90.00000 55.00000 100.0000 55.00000 110.0000
0.0 0.0 0.0 0.0
55.00000 120.0000 55.00000 130.0000 55.00000 140.0000 55.00000 150.0000
0.0 0.0 0.0 0.0
55.00000 160.0000 55.00000 170.0000 55.00000 180.0000 55.00000 190.0000
0.0 0.0 0.0 0.0
55.00000 200.0000 55.00000 210.0000 55.00000 220.0000 55.00000 230.0000
0.0 0.0 0.0 0.0
55.00000 240.0000 55.00000 250.0000 55.00000 260.0000 55.00000 270.0000
0.0 0.0 0.0 0.0
55.00000 280.0000 55.00000 290.0000 55.00000 300.0000 55.00000 310.0000
0.0 0.0 0.0 0.0
55.00000 320.0000 55.00000 330.0000 55.00000 340.0000 55.00000 350.0000
0.0 0.0 0.0 0.0
55.00000 360.0000 55.00000 370.0000 55.00000 380.0000 55.00000 390.0000
0.0 0.0 0.0 0.0
55.00000 400.0000 55.00000 410.0000 55.00000 420.0000 55.00000 430.0000

0.0 0.0 0.0 0.0
55.000000 440.00000 55.000000 450.00000 55.000000 460.00000 55.000000 470.00000
0.0 0.0 0.0 0.0
55.000000 480.00000 55.000000 490.00000 55.000000 500.00000 55.000000 510.00000
0.0 0.0 0.0 0.0
55.000000 520.00000 55.000000 530.00000 55.000000 540.00000 55.000000 550.00000
0.0 0.0 0.0 0.0
55.000000 560.00000 55.000000 570.00000 55.000000 580.00000 55.000000 590.00000
0.0 0.0 0.0 0.0
55.000000 600.00000
0.0

Case 4:

0 -10 0 0 58 -10 58 0 58.34418 9.996141 58.55541 19.99088 58.63890 29.98408
0.0 0.0 0.0 0.0
58.59988 39.97561 58.44358 49.96532 58.17523 59.95308 57.80006 69.93875
0.0 0.0 0.0 0.0
57.32328 79.92219 56.75013 89.90327 56.08583 99.88184 55.33574 109.8561
0.0 0.0 0.0 0.0
54.50746 119.7992 53.60415 129.7394 52.63099 139.6765 51.59316 149.6104
0.0 0.0 0.0 0.0
50.49584 159.5411 49.34420 169.4683 48.14343 179.3919 46.89871 189.3118
0.0 0.0 0.0 0.0
45.61521 199.2278 44.29812 209.1399 42.95246 219.0488 41.58180 228.9662
0.0 0.0 0.0 0.0
40.19304 238.8791 38.79138 248.7875 37.38202 258.6912 35.97016 268.5901
0.0 0.0 0.0 0.0
34.56101 278.4840 33.15976 288.3728 31.77162 298.2564 30.40178 308.1345
0.0 0.0 0.0 0.0
29.05544 318.0072 27.73624 327.8862 26.43887 337.8542 25.18153 347.8161
0.0 0.0 0.0 0.0
23.96959 357.7717 22.80841 367.7211 21.70338 377.6640 20.65985 387.6002
0.0 0.0 0.0 0.0
19.68320 397.5297 18.77881 407.4522 17.95205 417.3678 17.20828 427.2761
0.0 0.0 0.0 0.0
16.55068 437.2128 15.98242 447.2406 15.51607 457.2605 15.15724 467.2725
0.0 0.0 0.0 0.0
14.91151 477.2763 14.78447 487.2719 14.78172 497.2590 14.90886 507.2376
0.0 0.0 0.0 0.0
15.17147 517.2075 15.57515 527.1685 16.12549 537.1205 16.82659 547.0442
0.0 0.0 0.0 0.0
17.68353 556.9439 18.70277 566.8342 19.88984 576.7150 21.25029 586.5862
0.0 0.0 0.0 0.0
22.78965 596.4475 24.51346 606.2988 26.42725 616.1400 28.53656 625.9709
0.0 0.0 0.0 0.0
30.84693 635.7915 33.36389 645.6015 35.97962 655.0088 38.77047 664.3248
0.0 0.0 0.0 0.0
41.76302 673.6308 44.96205 682.9268 48.37232 692.2126 51.99862 701.4880
0.0 0.0 0.0 0.0
55.84570 710.7530 59.91834 720.0074 64.22132 729.2511 68.75941 738.4840
0.0 0.0 0.0 0.0
73.53737 747.7060 78.25212 756.3653 82.91911 764.5553 87.78686 772.7363

0.0 0.0 0.0 0.0
92.85873 780.9082 98.13809 789.0710 103.6283 797.2246 109.3327 805.3688
0.0 0.0 0.0 0.0
115.2547 813.5036 121.3977 821.6290 127.7650 829.7448 134.3600 837.8509
0.0 0.0 0.0 0.0
141.1860 845.9472 148.2464 854.0337
0.0 0.0

Case 5:

visio5=[50.83246 51.49635 19.90502 51.98965 29.90602 52.30954 39.89791
9.934537 0.0 0.0 0.0 0.0
52.45585 49.88534 52.42756 59.88684 52.22283 69.89226 51.84095 79.87494
0.0 0.0 0.0 0.0
51.27964 89.85570 50.53694 99.82925 49.61147 109.7844 48.50119 119.7194
0.0 0.0 0.0 0.0
47.20392 129.6313 45.71734 139.5172 44.03862 149.3759 42.16668 159.1956
0.0 0.0 0.0 0.0
40.09956 168.9710 37.83152 178.7124 35.36286 188.4021 32.69232 198.0313
0.0 0.0 0.0 0.0
29.81540 207.6030 26.72803 217.1161 23.43194 226.5525 19.92513 235.9068
0.0 0.0 0.0 0.0
16.20009 245.1870 12.25949 254.3747 8.101542 263.4638 3.727004 272.4440
0.0 0.0 0.0 0.0
0.8627068 281.3130 5.665326 290.0737 10.67637 298.7247 15.89137 307.2646
0.0 0.0 0.0 0.0
21.30584 315.6921 26.91530 324.0058 32.68911 332.1680 38.61655 340.1741
0.0 0.0 0.0 0.0
44.72217 348.0643 51.00162 355.8374 57.45055 363.4921 64.06460 371.0269
0.0 0.0 0.0 0.0
70.83942 378.4405 77.74387 385.7039 84.74307 392.7886 91.88662 399.7509
0.0 0.0 0.0 0.0
99.17032 406.5896 106.5900 413.3034 114.1414 419.8910 121.8203 426.3509
0.0 0.0 0.0 0.0
129.6157 432.6765 137.4691 438.8253 145.4355 444.8449 153.5109 450.7340
0.0 0.0 0.0 0.0
161.6911 456.4913 169.9721 462.1155 178.3497 467.6053 186.8198 472.9596
0.0 0.0 0.0 0.0
195.3655 478.1693 203.9924 483.2395 212.6993 488.1708 221.4821 492.9619
0.0 0.0 0.0 0.0
230.3368 497.6116 239.2592 502.1186 248.2453 506.4815 257.3239 510.7142
0.0 0.0 0.0 0.0
266.4890 514.8128 275.7064 518.7614 284.9717 522.5590 294.2809 526.2040
0.0 0.0 0.0 0.0
303.6297 529.6953 313.0141 533.0315 322.4728 536.2255 332.0754 539.2967
0.0 0.0 0.0 0.0
341.7018 542.2035 351.3475 544.9446 361.0083 547.5186 370.6796 549.9241
0.0 0.0 0.0 0.0
380.3572 552.1598 390.0734 554.2318 400.0099 556.1715 409.9390 557.9278
0.0 0.0 0.0 0.0
419.8560 559.4992 429.7561 560.8843 439.6347 562.0815 449.4868 563.0895
0.0 0.0 0.0 0.0