DYNAMIC SIMULATION OF COMPUTED-TORQUE CONTROLLERS FOR A WHEELED MOBILE ROBOT AUTONOMOUS NAVIGATION IN OUTDOOR ENVIRONMENTS

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ABSTRACT
This paper describes the development of PD and PID Computed-Torque (CT) controllers for autonomous navigation of a Wheeled Mobile Robot (WMR) in outdoor environments. WMRs have complex and nonlinear dynamics, which make their modeling and control difficult. The controllers select the suitable control torques, so that the WMR follows the desired path produced from a navigation algorithm described in a previous paper. PD CT and PID CT controllers were developed and tested under different motion trajectories. Software was developed to simulate the performance and efficiency of the controllers. Simulation results verified the effectiveness of the controllers. The significance of this work lies in the development of CT controllers for WMR navigation, instead of robot manipulators. These CT controllers will facilitate the use of WMRs in many applications.

INTRODUCTION

This paper presents the development of a proportional-plus-derivative (PD) CT and a proportional-plus-integral-plus-derivative (PID) CT controllers. A dynamic simulation, based on a framework developed by Lewis, et al. (1999), was conducted after modifying it to suit the navigation of a WMR. The simulation software takes, as input, the desired robot path from the navigation algorithm described in a previous paper by the authors (Alhaj Ali and Hall, 2002). The simulation software produced the suitable control torques. This simulation was developed by using matlab and C++.

Different controllers had been developed for the motion of robot manipulators. However, not until recently has there been an interest in moving the robot itself, not just its manipulators. Thus, WMR motion control is a new research area.

Based on empirical practice, Shim and Sung (2003) proposed a WMR asymptotic control with driftless constraints by using the WMR kinematic equations. The trajectory control of a wheeled inverse pendulum type robot had been discussed by Yun-Su and Yuta (1996).

Rajagopalan and Barakat (1997) developed a computed torque control scheme for Cartesian velocity control of WMRs. A discontinuous stabilizing controller for WMRs with nonholonomic constraints was presented by Zhang and Hirschorn (1997). A path tracking problem was formulated by Koh and Cho (1999) for a mobile robot, which follows a virtual target vehicle moving with a specified velocity exactly along the path. Zhang, et al. (2003) employed dynamic modeling to design a tracking controller for a differentially steered mobile robot subject to wheel slippage and external loads. A sliding mode control was used to develop a trajectory tracking control in the presence of bounded uncertainties (Corradini and Orlando, 2001). Jiang, et al. (2001) developed a model-based control design strategy dealing with global stabilization and global tracking control for the WMR kinematic model.

A fuzzy logic controller had also been tried for navigating WMRs. Montaner and Ramirez-Serrano (1998) developed a fuzzy logic controller, which can deal with the sensors inputs uncertainty and ambiguity for direction and velocity maneuvers. Toda, et al. (1999) employed a sonar-based mapping of crop rows and fuzzy logic control-based steering for navigating a WMR in an agricultural environment.

PD AND PID CT CONTROLLERS

This section describes the development of PD and PID CT controllers for WMR autonomous navigation in outdoor environments. To provide focus for the presented model, a particular configuration will be addressed: the Bearcat III robot, which has two driven fixed wheels and a caster. Bearcat III was developed at the Center for Robotics Research, at the University of Cincinnati.

A simplified kinematic and dynamic model is derived from the Newton-Euler method by considering the velocity along the x and y axes and the angular velocity, with the robot center of mass as a reference point.

$$M(q)\ddot{q} + J(q,\dot{q})\dot{q} + F = \tau$$

where:

$$q = \begin{bmatrix} x_c \\ y_e \\ \theta \end{bmatrix}, \quad F = \begin{bmatrix} \dfrac{-f_n e r}{d} \\ \dfrac{-f_n e r}{d} \end{bmatrix}, \quad \tau = \begin{bmatrix} \tau_r \\ \tau_l \end{bmatrix}$$

$$M(q) = \begin{bmatrix} \dfrac{(mr^2\cos\theta + 2J_0\cos\theta)}{2r} & \dfrac{(mr^2\sin\theta + 2J_0\sin\theta)}{2r} & \dfrac{(mr^2 ed\sin^2\theta - mr^2 ed\sin\theta\cos\theta + J_c r^2 + 2J_0 d^2}{2rd} \\ \dfrac{(mr^2\cos\theta + 2J_0\cos\theta)}{2r} & \dfrac{(mr^2\sin\theta + 2J_0\sin\theta)}{2r} & \dfrac{(mr^2 ed\sin^2\theta - mr^2 ed\sin\theta\cos\theta - J_c r^2 - 2J_0 d^2}{2rd} \end{bmatrix}$$

$$J(q,\dot{q}) = \begin{bmatrix} \dfrac{-J_0\dot{\theta}\sin\theta}{r} & \dfrac{J_0\dot{\theta}\cos\theta}{r} & \dfrac{-mre\dot{\theta}\cos\theta(\sin\theta + \cos\theta)}{2} \\ \dfrac{-J_0\dot{\theta}\sin\theta}{r} & \dfrac{J_0\dot{\theta}\cos\theta}{r} & \dfrac{-mre\dot{\theta}\cos\theta(\sin\theta + \cos\theta)}{2} \end{bmatrix}$$

x_c : is the x- coordinate of the robot's center of mass.

y_e : is the y- coordinate of the center of the wheel axle.

θ : is the orientation angle of the robot.

f_n : is the resultant normal force.

e : is the distance between the robot's center of mass and the center of the wheel axle.

r : is the radius of the fixed wheel.

d : is half the width of the robot.

τ_r : is the right motor torque, acting on the right wheel.

τ_l : is the left motor torque, acting on the left wheel.

m : is the robot's mass, excluding the wheels.

J_0 : $J_0 = J_w + m_w r^2$, where J_w is the inertia of the wheel, and m_w is the mass of the wheel.

J_c : is the robot's inertia, excluding the wheels.

(1)

For more details, please refer to Alhaj Ali (2003).

The robot dynamics can be written as follows (Lewis, et al., 1999):

$M(q)\ddot{q} + N(q,\dot{q}) = \tau$, where $N(q,\dot{q})$ represents the nonlinear terms, $N(q,\dot{q}) = J(q,\dot{q})\dot{q} + F$ (2)

The objective of a motion controller is to move the robot along a desired motion trajectory $q_d(t)$. The actual motion trajectory is defined as $q(t)$. The tracking error, in this case, is defined as (Lewis, et al., 2000):

$$e(t) = q_d(t) - q(t) \quad (3)$$

The Brunovsky canonical form is developed by differentiating $e(t)$ twice and writing it in terms of the state x (Lewis, et al., 2000):

$$\frac{d}{dt}\begin{bmatrix} e \\ \dot{e} \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}\begin{bmatrix} e \\ \dot{e} \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} u, \quad (4)$$

where $u \equiv \ddot{q}_d + M^{-1}(q)(N(q,\dot{q}) - \tau)$, $x = \begin{bmatrix} e^T & \dot{e}^T \end{bmatrix}^T$

To develop the CT controller, a linear system design procedure will be used to select a feedback control, $u(t)$, that stabilizes the tracking error equation. The torques needed for the motors will then be computed by using the inverse of the dynamic equation for the WMR:

$$\tau = M(q)(\ddot{q}_d - u) + N(q,\dot{q}) \quad (5)$$

A PD feedback for $u(t)$, with a derivative gain matrix, K_v, and a proportional gain matrix, K_p, produces the PD CT controller with the motor torques (Lewis, et al., 2000):

$$\tau = M(q)(\ddot{q}_d + K_v\dot{e} + K_p e) + N(q,\dot{q}), \quad (6)$$

which has the tracking error dynamics $\ddot{e} = -K_v\dot{e} - K_p e$

The PD CT controller described above can easily be adjusted to a PID CT controller by adding an integrator gain matrix, K_i, to $u(t)$ (Lewis et al., 2000):

$$\tau = M(q)(\ddot{q}_d + K_v\dot{e} + K_p e + K_i(\int e)) + N(q,\dot{q}), \qquad (7)$$

which has the tracking error dynamics $\ddot{e} = -K_v\dot{e} - K_p e$

Simulation of the PD CT controller for WMR navigation

The simulation program has the following main components:

- The first component computes the desired WMR trajectory from the input of the navigation algorithm.
- The second component calculates the controller input by first calculating the tracking error between the desired trajectory and the actual trajectory and then computing the inertia term and the nonlinear term from the WMR dynamic model described in Eq. 1. Finally, the motor torques are calculated by using Eq. 6.
- The third component calculates the new position of the WMR, using the state-space equation, $\dot{x} = f(x,u)$, in which the state-space position/velocity form is used (Lewis, et al., 2000):

$$x \equiv \begin{bmatrix} q^T \\ \dot{q}^T \end{bmatrix}, \quad \dot{x} = \begin{bmatrix} \dot{q} \\ -M^{-1}(q)N(q,\dot{q}) \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1}(q) \end{bmatrix}\tau \qquad (8)$$

This is used to update the WMR's actual position. Since $M(q)$ is not square, the Moore-Penrose generalized inverse is used.

Several experiments had been conducted on the PD CT simulation software; the robot parameters were from Bearcat III. Various controller parameters were tested by sinusoidal desired motion trajectories.

Experiments show that k_p and k_v need to be different for each component of the motion trajectory, q. Small positive values are required to obtain good results. The gain parameters, k_{p1} =2, k_{v1} =1, k_{p2} =0, k_{v2} =10, k_{p3} =2, and k_{v3} =1, give good results, as shown in the following figures. The tracking errors are in the range 0.00-0.04, as shown in Fig. 1. The desired versus the actual motion trajectory is shown in Fig. 2. As shown in the figures, the tracking errors are zero for both x and θ, with the actual motion trajectory following the desired motion trajectory for these two components. For y, the error oscillates around 0.02.

The selection process of proper parameters for the controller is a challenging task, since six parameters must be changed in each trial. Although x, y, and θ are independent of each other, they are, nevertheless, used to update the new values of the actual motion trajectories. Hence, they have an impact upon each other. Therefore, keeping the same controller parameters for a particular motion trajectory component does not imply that the actual motion trajectory will remain the same.

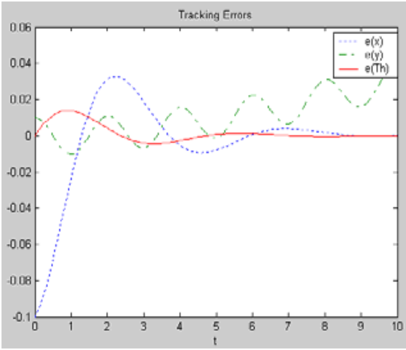

Figure 1: PD CT controller tracking errors versus time.

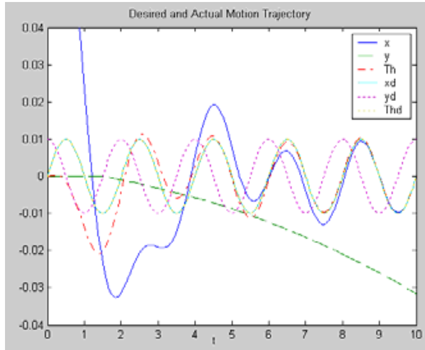

Figure 2: Desired versus actual motion trajectories.

Simulation of the PID CT controller for WMR navigation

The PID CT controller simulation program for WMR navigation was developed the same way as that of the PD CT controller, except for the equation for calculating the motor torques, where Eq. 7 was used.

Several experiments had been conducted on the PID CT simulation software, and different trajectories and controller parameters were tried.

The results of the experiments show that the controller parameters need to be small positive numbers to obtain good results. It is also noteworthy that increasing k_p and fixing k_v and k_i, or increasing k_v and fixing k_p and k_i, reduces the tracking errors of θ and x, while it increases the tracking error of y. However, there is a limit to this increase, which is about 10.

Using very large, or zero, values for k_p, k_v, or k_i is not recommended. Additionally, the value of k_i must not be too large, as a condition for having a stable tracking error.

k_p=2, k_v=1, and k_i=1 give very reasonable results, as shown in the following figures. The tracking error for θ is zero. For x, it oscillates around zero. For y, it starts at zero and increases to 0.35, as shown in Fig. 3. The desired versus the actual motion trajectories are shown in Fig. 4.

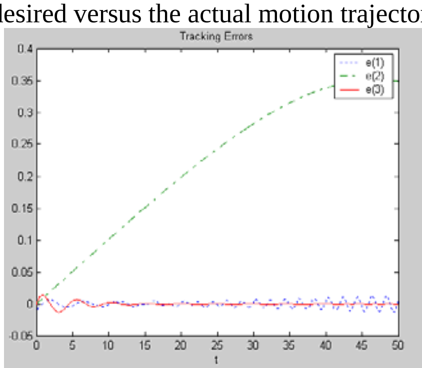

Figure 3: PID CT controller tracking errors versus time.

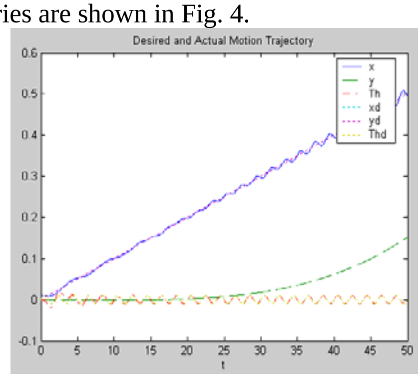

Figure 4: Desired versus actual motion trajectories.

CONCLUSIONS

In this paper, the development of a PD CT and PID CT controller for WMR autonomous navigation in an outdoor environment is presented. The controllers select suitable control torques, so that the WMR will follow the desired path from a navigation algorithm described in a previous paper. The controllers are tested under various control parameters and motion trajectories. Simulation results show that, for a PD CT controller, the gain matrices need to be different for each component of the motion trajectory; small positive values are needed to get good results. Simulation results for a PID CT controller show that the gain matrices also need to be small positive numbers. k_p =2, k_v =1, and k_i =1 produce reasonable results. Selecting the proper parameters for the controller is a challenging task, since there are too many parameters involved.

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