Abstract

The coupled oxygen transport in the avascular wall of a coronary artery stenosis is studied numerically by solving the convection-diffusion equations. Two geometries replicating stenosis before and after percutaneous transluminal coronary angioplasty (PTCA) are used for the analysis. The results are compared to evaluate the effect of the degree of stenosis on oxygen transport. Important physiological aspects, such as oxygen consumption in the wall, oxygen carried by the hemoglobin, non-Newtonian viscosity of the blood, and supply of oxygen from the vasa vasorum are included. The results show that the $P_{O_2}$ in the medial region of the arterial wall is $\sim 10$ mmHg. The oxygen flux to the wall increases in the flow acceleration region, whereas it decreases at the flow reattachment zone. Near the location of flow separation, there is a small rise followed by a sharp fall in the oxygen flux. The drop in the oxygen flux to the wall at the point of flow reattachment for pre-PTCA stenosis is four times that for post-PTCA stenosis. The minimum $P_{O_2}$ in the avascular wall, $P_{O_2,\text{min}}$, at this location decreases to $\sim 6.0$ and $\sim 4.2$ mmHg for post- and pre-PTCA stenosis, respectively. The drop in $P_{O_2,\text{wall}}$ and $P_{O_2,\text{min}}$ at the point of flow reattachment for pre-PTCA is $\sim 2$ times that for post-PTCA stenosis. Thus, the present study quantifies the oxygen transport to the arterial wall before and after cardiovascular intervention.

Keywords: Percutaneous transluminal coronary angioplasty (PTCA); Stenosis; Oxygen transport; Arterial wall; Oxygen consumption

1. Introduction

The large number of experimental and numerical work by various researchers over last four decades has suggested a link between atherosclerosis, hypoxia and lumen-arterial wall oxygen transport (Back, 1976; Crawford et al., 1983; Moore and Ethier, 1997; Rappitsch and Perktold, 1996; Schneiderman et al., 1974). Recently, Vaidya et al. (2005) from our research group have shown that the presence of the residual stenosis even for the post-percutaneous transluminal coronary angioplasty (PTCA) scenario has a significant effect on the supply of oxygen to the arterial wall. The detailed literature review in Vaidya et al. (2005) has not been included here. However, without analyzing and quantifying the pre-PTCA scenario, and comparing it with the post-PTCA case (Vaidya et al., 2005), the literature remains incomplete. In fact, from a physiological perspective it is very important to know the oxygen concentration for pre-PTCA condition. Therefore, the objective of the present study is to quantify the pathophysiological condition of the arterial wall before PTCA and compare this result with our post-PTCA results that have been published recently. Hence, two geometries representing pre- and post-PTCA coronary artery stenosis are compared here to evaluate the effect of the degree of stenosis on the oxygen transport.

2. Method

2.1. Geometry

Figs. 1A and B show the geometries used for the analysis in pre- and post-PTCA coronary artery stenosis, respectively. The dimensional details for the plaque are taken from the in vico data set of Wilson et al. (1988).
from a group of 32 patients undergoing PTCA and as well as from our previous analysis for the post-PTCA case (Vaidya et al., 2005). It is an axisymmetric artery with 90% (pre-PTCA) and 64% (post-PTCA) area stenosis at the throat region of stenosis. Zemplenyi et al. (1989) showed by experimental measurements that the vasa vasorum grows in the plaque as a counter mechanism to the hypoxia. Hence, the thickness of the avascular region is 300 m along the complete axial length. In the avascular wall region, the diffusion velocities are an order of magnitude greater than the convective velocity (Back, 1975). Hence, the oxygen transport in the avascular region is mainly by diffusion, and the wall is considered to be rigid. Since pulsatility has been shown to have negligible effect on oxygen transport in the near-wall region (Schneiderman et al., 1982), only steady flow is considered. The velocity field which determines the oxygen concentration distribution is established by solving the continuity and momentum conservation equations. The oxygen concentration in the blood is directly proportional to the partial pressure of oxygen, $P_{O_2}$.

### 2.2. Oxygen concentration equations

Taking into account the total oxygen carried by the hemoglobin and that which is dissolved in plasma, the oxygen mass conservation equation in the lumen becomes

$$\frac{\partial}{\partial t}(c + \gamma) + u \frac{\partial}{\partial z}(c + \gamma) + \frac{\partial}{\partial r}(c + \gamma) = D_b \left( \frac{\partial^2 c}{\partial r^2} + \frac{\partial c}{\partial r} + \frac{\partial^2 c}{\partial z^2} \right),$$

where $c = (zP_{O_2})$ is the oxygen concentration in mlO/mlblood, $\gamma$ is the solubility coefficient for oxygen, $z$ is the oxygen carried by the hemoglobin, and $D_b$ is the oxygen diffusivity in blood ($1.0 \times 10^{-5}$ cm$^2$/s).

The oxygen concentration conservation equation in the wall region is

$$\frac{\partial c}{\partial t} = \nabla(D_w \nabla c) - \dot{q},$$

where $\dot{q}$, the constant volumetric consumption rate of oxygen by the cells within the wall region, is $1.3 \times 10^{-4}$ mlO/mlblood/s (Crawford et al., 1983), and $D_w$ is the oxygen diffusivity in the wall ($1.0 \times 10^{-5}$ cm$^2$/s).

### 2.3. Boundary conditions

A constant flow rate of 50 ml/min (basal), with a parabolic profile for axial velocity, is applied at the inlet. The Reynolds number, based on the inlet diameter, is 100. The Schmidt number is 3500. Uniform concentration of oxygen, corresponding to normal blood $P_{O_2}$ of 95 mmHg, is applied at the inlet (Jurrus and Weiss, 1977). At vasa vasorum, oxygen concentration of 45 mmHg is applied (Crawford et al., 1983).

### 3. Results

The concentration and flux of oxygen shows wide variations in different regions of the stenosis as well as between pre- and post-PTCA stenosis. Our previous analysis (Vaidya et al., 2005) has explained the results for the post-PTCA case in detail; thus, the current study primarily compares the new results from pre-PTCA with those for post-PTCA stenosis.

#### 3.1. $P_{O_2,w}$ along the axial length

Fig. 2 shows that there are significant differences between pre- and post-PTCA stenosis primarily in (1) partial pressure of oxygen at the lumen-endothelium interface, $P_{O_2,w}$, upstream of the converging section (arrow #1); and (2) $P_{O_2,w}$ at the flow reattachment point (arrow #2) and recirculation length (arrow #3) downstream of the stenosis. At the start of the converging region of the stenosis, the $P_{O_2,w}$ sharply increases for pre-PTCA (Fig. 2A) as compared to post-PTCA (Fig. 2B). This is due to the increased magnitude of the velocity gradients caused by greater area occlusion for pre-PTCA stenosis. At $z = 4.5$ cm, i.e., at the point of flow reattachment for pre-PTCA stenosis, the $P_{O_2,w}$ reaches to 57 mmHg, due to the reduced convective flow near the wall. The drop in $P_{O_2,w}$ at the location of reattachment for pre-PTCA stenosis is two times ($=[75–57]$ mmHg/[73–64] mmHg) greater than that for post-PTCA stenosis. As expected, the location of flow reattachment ($z = 4.5$ cm) for pre-PTCA stenosis is further downstream from the end of the stenosis than that for post-PTCA stenosis ($z = 2.4$ cm).
3.2. Oxygen flux to the wall

Oxygen flux (Fig. 3) has demonstrated significant changes at the start of the converging section, the flow separation points and the flow reattachment points. At the start of this converging section (arrow #1), the flux to the wall reduces significantly. This is because the increase in $P_{O_2;w}$ (Fig. 2) causes significant reduction in the numerator of the concentration gradient term, $\frac{\partial c}{\partial r}$, as compared with the reduction in the denominator. This, in turn, reduces the overall radial concentration gradient, and subsequently the oxygen flux to the wall. The drop in the oxygen flux at the start of the converging section for pre-PTCA stenosis (Fig. 3A) is almost 10 times greater than that for post-PTCA stenosis (Fig. 3B). For pre-PTCA stenosis (Fig. 3A), at about $z = 1.68$ cm, the flow separates and the reduction in the convective flux causes the oxygen flux to drop sharply. It reaches $8.06 \times 10^{-7}$ mlO/cm²/s at $z = 1.81$ cm (arrow #2). In contrast, the flow separates at $z = 1.95$ cm (arrow #2) and the flux drops to $1.79 \times 10^{-6}$ mlO/cm²/s for the post-PTCA case (Fig. 3B). The magnitude of negative wall shear stress is also close to zero at this location. At the point of flow reattachment (arrow #3), for pre-PTCA stenosis, at $z = 4.5$ cm, the oxygen flux decreases sharply to $1.65 \times 10^{-6}$ mlO/cm²/s due to reduced convective flow near the wall as compared to $2.12 \times 10^{-6}$ mlO/cm²/s at $z = 2.4$ cm (arrow #3) for post-PTCA stenosis. Stangeby and Ethier (2002), in a recent numerical study, also reported that the mass flux to the wall increases in the acceleration region and decreases in the deceleration region reaching the minimum at the point of flow separation.

Fig. 2. $P_{O_2;w}$ along the axial length between pre-PTCA (A) and post-PTCA (B) stenosis arteries.
3.3. $P_{O_2,\text{min}}$ along the axial length

Since the $P_{O_2,w}$ and the oxygen flux show relatively higher magnitude for pre-PTCA stenosis from the converging to the diverging section, the minimal partial pressure of oxygen, $P_{O_2,\text{min}}$, in the avascular wall also increases as compared to the post-PTCA. Nevertheless, it is noteworthy that the drop in $P_{O_2,\text{min}}$ at the location of flow reattachment for pre-PTCA stenosis (Fig. 4A) is almost two times ($=[9.5-4.2]\text{mmHg}/[8.4-5.75]\text{mmHg}$) larger than that for post-PTCA stenosis (Fig. 4B). At this location, the oxygen flux to the wall drops significantly as shown in Fig. 3A (arrow #1). This region may be especially prone to hypoxic injury.

4. Discussion

The important conclusion from this study is that the stenosis in both pre- and post-PTCA conditions has significant impact on the oxygen transport to the wall. Comparatively, the stenosis in pre-PTCA condition shows larger variations in oxygen flux, $P_{O_2,w}$ and $P_{O_2,\text{min}}$. The value of $P_{O_2,\text{min}}$ at the axially central location of the throat for pre-PTCA stenosis is higher than that for post-PTCA stenosis. But the in-growth of easa vasorum may not always be sufficient in pre-PTCA stenosis to maintain the avascular thickness of 300 µm. In such a case, the $P_{O_2,\text{min}}$ may decrease to a low value compared with that for post-PTCA stenosis, making the medial region of the wall susceptible to hypoxic injury. It is also evident that oxygen flux is affected by the magnitude and gradients of flow velocity. The locations of the core of the recirculation show lower oxygen concentration. Near the location of flow separation the oxygen flux in pre-PTCA stenosis drops to $8.06 \times 10^{-7}\text{mlO/cm}^2/\text{s}$ as compared to $1.81 \times 10^{-6}\text{mlO/cm}^2/\text{s}$ for post-PTCA stenosis. Similarly, at the flow reattachment location the $P_{O_2,\text{min}}$ drops to 4.2 mmHg for pre-PTCA stenosis as compared to 5.8 mmHg for post-PTCA stenosis. The location of flow reattachment for both pre- and post-PTCA stenosis where the value of $P_{O_2,\text{min}}$ drops to a very low value may become a site of initiation for a secondary lesion. Thus, these
conclusions from the quantification of oxygen transport will become significant for helping to understand the locations of impaired oxygen transport after cardiovascular treatments like stent placement.

Conflict of interest

We declare that we have no conflict of interest.

References


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