Estimated Flow Resistance Increase in a Spiral Human Coronary Artery Segment

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Coronary flow estimates were made for a spiral coronary artery segment (identified from a post-mortem replica casting) by using a modified Dean number based on the approximate coil radius of curvature, as suggested earlier. The estimates were found to correlate experimental pressure drop data for helical coiled tubes. Over a physiological range of mean Reynolds numbers from 100 to 400 for blood flow through main coronary arteries, estimates of the flow resistance increase relative to a straight lumen segment ranged from about 20 to 80 percent, and were of similar magnitude to those found in a flow study in a sinuous coronary vessel segment with no spiral. [SO148-0731(00)01706-4]

Introduction

The main coronary artery system of man, which courses the epicardium of the heart, is anatomically very complex. Common features are vessel curvature, branching, and taper. Some vessels appear sinuous or tortuous in shape, and a few appear kinked. Also, because of ventricle motion, there is vessel bending.

Coronary arteriography, as it is usually practiced with single plane or biplane views, reveals curved portions of vessels (e.g., see Gorlin [1]) but the actual configuration of curved vessel segments is less clear because of their three-dimensional orientation. A case in point is seen in Fig. 1 by the orthogonal x-ray views of a replica casting of a segment of a coronary artery of a human cadaver obtained from the USC School of Medicine through the Willed Body Program. The casting technique has been described previously (Crawford et al. [2] and Cho et al. [3]). In the top view, the vessel curves in one direction, then in the other direction and then again in the initial direction, forming a gradual kink. In the side view, the vessel appears to have a pronounced kink in the curvature region. However, neither of these orthogonal views indicate the actual segment shape that is evident by looking along the axis of the vessel, which reveals a spiral or twisted shape as seen in the end view in Fig. 1. The spiral angle is through one turn (360 deg) with the longitudinal axis at the end of the spiral displaced from the inlet of the spiral by about two vessel diameters. The axial length of the spiral segment is about 40 mm, and there is some taper as the vessel diameter changes from 4.0 mm to 3.5 mm. In this spiral segment, there is also a complex pattern of wall roughness characteristic of diffuse atherosclerotic disease, especially in the first 180 deg of the spiral along the outer curvature wall.

An example of vessel spiraling is also evident in the coronary angiogram shown in Fig. 2 of Vrints et al. [4] at the edge of the epicardium. We also have observed some coronary vessel spiraling in silicon rubber castings from other human cadavers. Reasons for the occurrence or development of vessel spiraling in some coronary vasculatures are not well understood.

The purpose of this initial study is to estimate the pressure loss and thus flow resistance through the spiral shape coronary segment. Our earlier in vitro flow studies in coronary artery castings of man focused on mainly straight segments with various degrees of "non-obstructive" diffuse atherosclerotic disease (Cho et al. [5], Back et al. [5], Cho et al. [6], and Back et al. [7]). Diffuse atherosclerosis is often difficult to detect from patient angiograms (e.g., Marcus et al. [8] and Wilson and Laxon [9]). More recently flow was studied in an S-shaped reversed curvature segment of an atherosclerotic coronary artery casting with mild curvature in the ventricle contour plane (Back et al. [10]). In this case there was no spiral in the sinuous coronary vessel segment studied. The background section gives pertinent discussion on curvature effects.

A range of physiological flow rates and corresponding Reynolds numbers Re for main coronary arteries from resting mean values to a factor of about four higher for exercise conditions was considered, i.e., Re from about 100 to 400, for which the flow is laminar.
Background-Curvature Effects

In reviewing the extensive literature on the effects of curvature on flow through tubes going back to the early work by Dean [11,12], and in particular the review by Berger et al. [13], including biomedical applications, and mention of subsequent studies and limitations on the understanding of flows in vessels with varying curvature, taper, and wall roughness by Back et al. [14,15], one finds much less information on flow in spiral tubes. About the only type of spiraling that has been investigated is that for helical coiled tubes used in engineering applications such as in heat exchangers. The helix angle $\phi$ is defined by

$$\tan \phi = \frac{b}{2a} = \frac{b}{4a}$$

(1)

In Eq. (1), $a$ is the radius of the coils measured from their longitudinal axis, and $b$ is the pitch or axial distance between successive coils. For helical coils, the radius of curvature of the coil, $r_c$, is given by the Frenet relation

$$r_c = a \left[ 1 + \left( \frac{b}{2 \pi a} \right)^2 \right]^{1/2}$$

(2)

While there are many papers on the difficult calculation of the three-dimensional flow field for laminar flow through a helically coiled tube (e.g., Tuttle [16], Kao [17], Germano [18], Wang [19], Murata et al. [20], and Truesdell and Adler [21]), there are few measurements of pressure drop over a wide range of geometric configurations and Reynolds numbers.

Mishra and Gupta [22] measured pressure drops for a fully developed laminar (and turbulent) flow through helically coiled smooth tubes of constant diameter over a large range of variables; coil to tube ratios, $a/r_w$, from 6.7 to 461; ratio of pitch to circumference of the coil, $b/2\pi a$, from 0 to 8 for which the helix angle, $\phi$, varied from 0 to 85 deg, and over a large range of Re from 120 to $10^5$. In the laminar flow regime, Mishra and Gupta [22] correlated their pressure data in terms of a modified Dean number, $\kappa_m$, by replacing the coil radius $a$ in the Dean number, $\kappa = \text{Re}(r_w/a)^{1/2}$, by the radius of curvature $r_c$ of the coil as earlier suggested by Truesdell and Adler [21] but thought to be restricted to relatively small pitch helical coils

$$\kappa_m = \text{Re} \left( \frac{r_w}{r_c} \right)^{1/2} = \text{Re} \left[ 1 + \left( \frac{b}{2 \pi a} \right)^2 \right]^{1/2} = \frac{\kappa}{1 + \left( \frac{b}{2 \pi a} \right)^2}$$

(3)

Their correlation is given by the following expression:

$$f_c = 1 + 0.033(\log_{10} \kappa_m)^4$$

(4)

where $f$ is the friction factor, and the ratio, $f_c/f_s$, gives the increase in frictional pressure drop in a curved tube because of secondary fluid motion due to centrifugal forces, compared to a straight tube. Here we use the Darcy friction factor

$$f = \frac{\Delta p_f}{\frac{1}{2} \rho u^2}$$

(5)

for which the Poiseuille relation for fully developed laminar flow through a straight tube is

$$f_s = \frac{\Delta p_f}{\frac{1}{2} \rho u^2} = \frac{64}{	ext{Re}}$$

(6)

The Fanning friction factor used by Mishra and Gupta [22] is 16/Re or a factor of 1/4 lower, because of the way in which the friction factors are defined.

### Table 1 Flow resistance increase with Reynolds number

<table>
<thead>
<tr>
<th>Re</th>
<th>$\kappa$</th>
<th>$\kappa_m$</th>
<th>$R_c/R_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>58</td>
<td>40</td>
<td>1.22</td>
</tr>
<tr>
<td>200</td>
<td>115</td>
<td>79</td>
<td>1.43</td>
</tr>
<tr>
<td>300</td>
<td>173</td>
<td>119</td>
<td>1.61</td>
</tr>
<tr>
<td>400</td>
<td>231</td>
<td>158</td>
<td>1.77</td>
</tr>
</tbody>
</table>

The increase in pressure drop $\Delta p_f$ for a curved tube compared to a straight tube $\Delta p_f$, i.e., the ratio $\Delta p_f/\Delta p_f$, is the same as the flow resistance ratio, $R_c/R_s$, for the same flow rate, so that for comparison purposes, Eq. (4) for a helical tube can also be written as

$$\frac{R_c}{R_s} = 1 + 0.033(\log_{10} \kappa_m)^4$$

(4')

From the Frenet relation, the effect of coil elongation (i.e., increasing $b/a$) is to decrease $R_c/R_s$. For further information on other friction factor ratio correlation equations for fully developed laminar flow in helically coiled tubes, see Manlapaz and Churchill [23].

### Results and Discussion

Estimates of the increase in flow resistance due to vessel spiraling were made by considering flow through a helical tube relative to a straight tube of the same length $\Delta s$, using the preceding relations. In this method, the radius of the spiral is about 8 mm for the first half turn (180 deg) and about 4 mm for the second half turn (180–360 deg), thus giving an average value of $a=6$ mm over the axial length, $b=40$ mm, of the one turn (360 deg). The vessel radius at the inlet $r_w=2.0$ mm. Thus, the effective helix angle, $\phi=59$ deg and the radius ratios ($r_w/a)=1/3$ and $(b/a)=6.7$. The radius of curvature of the coil, $r_c=12.8$ mm, and the ratio radius, $(r_w/r_c)=0.16$.

Over a physiological range of Reynolds numbers from 100 to 400 for blood flow through main coronary arteries, the estimates of flow resistance increase are given in Table 1. They range from about 20 to 80 percent, indicating moderate increases over the range of Re. These values are of similar magnitude to those found for steady flow measurements in a sinuous coronary vessel segment casting (Back et al. [10]), which varied from 14 to 56 percent over the same Re range.

Flow resistance increases would probably be somewhat higher for the rather “tight” spiral with ratio of coil to vessel radius, $a/r_w=3$, a value below the smallest helical coil ratio of 6.7 investigated by Mishra and Gupta [22] (e.g., see the brief mention of Farragia’s data [24] for laminar flow in helical coils by Ward-Smith [25]). Conversely, flow resistance increases would be somewhat less because flow through one turn is in the entrance region of a curved tube and not in the asymptotic fully developed flow region for which the correlations apply (e.g., see Back et al. [26]), for pressure measurements in a mildly curved femoral artery model.

Nevertheless, these estimates give the relative magnitude of flow resistance increases due to curvature effects expected in the spiral vessel segment. Because of the complexity of diseased coronary vessel anatomy, it is difficult to isolate the separate effects of vessel curvature, taper, and wall roughness on flow resistance. Clearly, more work is needed in this regard, including pulsatile flow simulation. Back et al. [10] found additional increases in pulsatile flow resistance above the steady flow values amounting to 15 percent at Re=100 and 30 percent at Re=400 for a
References

- Mal coronary arteries during cardiac surgery (Kajiya et al. [27]), using the Poiseuille relation as a reference datum to appraise the effect of vessel curvature is a reasonable approximation in flow simulations. This is also consistent with the moderate value of the frequency parameter $\alpha$ = 3 for normal heart rates, kinematic viscosity $\nu = 0.035$ cm$^2$/s for blood, and a vessel diameter of 4 mm (e.g., Lighthill [28]). Moderate pressure losses in curved regions given by $(\Delta p/\Delta x) = (R_c/R_r)(128\nu/\pi a^4)$, may occur in smaller diameter coronary vessels during hyperemic blood flow.

Nomenclature

- $a =$ radius of spiral or coil
- $A =$ lumen cross-sectional area
- $b =$ pitch or axial distance between successive coils
- $d =$ lumen diameter
- $f =$ Darcy friction factor = $(\Delta p/0.5\eta u^2)(d/\lambda x)$
- $p =$ mean pressure
- $Q =$ mean volume flow rate
- $R_c =$ radius of curvature of spiral segment
- $r_w =$ lumen radius
- $Re =$ Reynolds number based on inlet lumen diameter = $4Q/\pi rv d$
- $R_s =$ flow resistance for spiral segment
- $R_f =$ flow resistance for straight lumen
- $s =$ axial distance along spiral segment
- $u =$ axial velocity = $Q/A$
- $\alpha =$ frequency parameter = $(d/2)[(\omega r)^{1/5}$
- $\kappa =$ Dean number for curved pipe = $Re(r_c/la)^{0.5}$
- $\mu =$ modified Dean number = $Re(r_c/la)^{0.5}$
- $\mu =$ viscosity
- $v =$ kinematic viscosity = $\mu/\rho$
- $p =$ density
- $T =$ period of cardiac cycle
- $\psi =$ helix angle for helical coil
- $\omega =$ circular frequency = $2\pi/T$

Subscript

- $i =$ inlet condition

Superscript

- $(')$ = time average (mean) for cardiac cycle

Stability Analysis and Finite Element Simulation of Bone Remodeling Model

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Bone remodeling is widely viewed as a dynamic process—maintaining bone structure through a balance between the op-

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posed activities of osteoblast and osteoclast cells—in which the stability problem is often pointed out. By an analytical approach, we present a bone remodeling model applied to n unit-elements in order to analyze the stationary states and the condition of their stability. In addition, this theory has been simulated in a computer model using the Finite Element Method (FEM) to show a relationship between the bone remodeling process and the stability analysis. [S0148-0731(00)01806-9]

1 Introduction

The trabecular structure in bone is due in large part to the dynamic remodeling process controlled by mechanical function. The most striking examples of environmental adaptation are in professional athletes, who undergo increased bone size and density as a result of extreme skeletal loads, and in astronauts, who experience a decrease in bone density because of an absence of load in the weightless environment of space. The nature of the dependence between the form of bones and the load they carry was described by Wolff [1], who stated that "every change in the form of a living bone is followed by adaptive changes in its internal architecture and its external shape." Wolff’s work outlined the general scheme for remodeling, which has been studied qualitatively and quantitatively to date in order to explore the implications of this theory. This work raised considerable interest in other research fields, and several attempts to quantify the bone remodeling process have been reported in the literature [2,3]. In this paper, we will not focus our attention upon biological factors, because we assume that the mechanical function of bone material is of great importance and their influence is small or they are applied evenly over the region of interest. Moreover, we do not substantially interact with the stress-mediated response of the tissue. The hypothesis that osteocytes act as sensors of a mechanical signal and regulators of bone mass by mediating the actor cells—the osteoclasts and the osteoblasts—has often been advanced [4,5]. However, the mechanisms involved in the regulation of these cells and the feedback mechanism by which the bone tissue senses the change in load environment and initiates the deposition or resorption of bone tissue are not completely understood. Consequently, several attempts to quantify the bone remodeling process have been reported in the literature by using density remodeling rate equations as constitutive equations [6] that relate the rate of bone tissue deposition and resorption to some measure of the mechanical loading on the bone termed "stimulus." It must be emphasized that different techniques have been employed in the computational modeling of this process, principally using the FEM [5,7–12]. In this approach, the models are all based on the principle that bone remodeling is induced by a local mechanical signal influenced by stress, strain, or strain energy. The stability of bone remodeling models has been assessed in a number of studies [9,13–16]. The authors indicated that the origin of the unstable behavior was the remodeling rule, rather than the details of the numerical approximations. However, it must be emphasized that Jacobs et al. [9] conclude that certain aspects of unstable behavior can be ascribed to the numerical approximation rather than the remodeling formulation itself.

We generalize the analytical work performed by Weinans et al. [13] and we apply the modified remodeling scheme of Mullender et al. [5]. We consider n unit elements [17] that can contain one sensor (osteocyte). The microstructural stimulus is taken as the strain energy divided by volumetric density raised to an exponent as in Harrigan and Hamilton [14], and in which an analytical solution for the stationary states is possible under some assumptions regarding the manner of loading for a region of bone. We show that the exponential character of the bone remodeling stimulus taken here does not change the necessary condition for a stable remodeling scheme also arrived at by Harrigan and Hamilton [14], who used variational methods applied to finite element remodeling bone models [18]. Note that Capello et al. [15] employed a general approach based on Lyapunov’s method to obtain the same necessary and sufficient condition for the global asymptotic stability of a specific bone remodeling theory. In order to show a relationship between the bone remodeling process and the stability of the overall structure of bone, a two-dimensional FEM model of a square plate was constructed and analyzed.

2 Remodeling Rate Equation

Cowin and Hegedus [6] proposed a class of models in which, at each point in the bone, the time rate of change of bone density is equated to a function that depends upon the density $\phi$, and upon the mechanical loading at a point in the bone. Consequently, the rate of change in the apparent density of the bone can be described by the normalized law, which was introduced by Mullender et al. [5] and expressed as:

$$\frac{\partial \phi_i(M,t)}{\partial t} = \eta \Psi(M, \phi, t) \quad \text{with} \quad \phi_{\text{min}} \leq \phi \leq \phi_{\text{max}}$$

(1)

where $\eta$ is a time constant regulating the rate of the process, $\phi_{\text{max}}$ is the density of cortical bone, $\phi_{\text{min}}$ is the density of completely resorbed bone. A modified spatial influence function $\Psi$, which depends on a particular stimulus at location $M$ in the trabecular bone, is introduced by adding the exponent parameter $\beta$.

$$\Psi(M, \phi, t) = \sum_{i=1}^{N} e^{-d(i,M)/d_0} \left[ \frac{U_i}{\phi} - U_0 \right]$$

(2)

where $U_i$ is the specific strain energy, $d(i,M)$ is the distance between sensor $i$ ($N$ is the total number of sensors) and the material position $M$, and $d_0$ is a parameter limiting the influence zone while $(U_i/\phi - U_0)$ is the difference between the stimulus and the reference signal.

For simplicity, we assume that the behavior of the bone is elastic, isotropic, and inhomogeneous. We assume a material Poisson ratio that is independent of density, and a power law relationship between density and the elastic modulus [19]

$$E(M,t) = E_0 \phi^\alpha(M,t)$$

(3)

where $E_0$ and $\alpha$ are experimentally derived parameters.

3 Stability Condition

We have extended the analytical study performed in the case of two unit-elements [13] by considering that: (a) the geometry is discretized with $n$ unit-elements and each element with area $A$ may contain in its center one sensor (Fig. 1), (b) the osteocytes act as sensors of a mechanical signal or mechano-receptors. The maximal number of sensors is $m(m \leq n)$ and $I_k (1 \leq k \leq m)$ is the set numbers of the sensors. Referring to Eqs. (1) and (2), we have the following general feedback equation for the $n$ unit-elements:

$$\frac{\partial \phi_i}{\partial t} = \eta \sum_{j=1}^{m} e^{-d(i,j)/d_0} \left[ \frac{U_j}{\phi^b_{i,j}} - U_0 \right]$$

(4)

where $d(i,j) = D|j-i|$ is the distance between elements $i$ and $j$.

Fig. 1 Geometry and loading configuration of the model with $n$ unit-element
scheme using constant time steps theory and a time stepping algorithm. The classical forward Euler results from the integration points. The model configuration is

\begin{equation}
\phi_i(t+\Delta \tau) = \phi_i(t) + \Delta \tau \Psi(\phi_i, t).
\end{equation}

On the other hand, the convergence behavior of this numerical model was investigated choosing the objective function \( \Gamma_i \) for each finite element \( i \) as:

\begin{equation}
\Gamma_i = \left| \frac{U_i}{\phi_i} - U_0 \right| \quad (i \text{ is not summed})
\end{equation}

where \( U_i \) is the specific strain energy for the element \( i \).

The value of \( \Gamma_i \) is calculated after each iteration and indicates the extent to which the objective is reached in each element. It must be noted that in those elements in which the bone resorbs completely (\( \phi = \phi_{\text{min}} = 0.01 \)) or in which cortical bone is reached (\( \phi = \phi_{\text{max}} = 1.74 \)), which corresponds to the Young’s Modulus value equal to 17000 MPa, the remodeling process stops, hence the objective \( U_i / \phi_i = U_0 \) will not be met.

A simple two-dimensional model of a unit square plate was constructed (Fig. 2). A mesh of 40 x 40 first-order quadrangular elements was made. The stresses and strains in the locations of the sensors were respectively calculated by linear interpolation from the stresses in the Gauss points integration and from the strains in the nodal points. The time step \( \Delta \tau \) in the Euler integration for the remodeling Eq. (7) was taken arbitrarily as 1.0 Time Units (TU) and the response to a uniform compression load for \( P = 10 \) N was chosen. Thus, a minimum value \( d_{\phi} = 0.025 \) mm was assumed, limited by the size of the element. The initial condition was assumed to be a uniform apparent density distribution. The reference signal and the time constant regulating the response were respectively \( U_0 = 0.04 \) MPa and \( \eta = 1 \) (MPa-TU)\(^{-1} \). The results take the form of the density as a function of different values of the parameters described above at 200 TU. This time corresponds to the value when the density does not change for any element and when the objective function (8) tends to 10\(^{-6} \). On the other hand, the well-known results in the stable case \( \alpha < \beta \) are omitted for the purpose of brevity. Indeed, we have retrieved the analytical results given by Eq. (5) for any initial density \( \phi_0 \). As example, taking \( \alpha = 3 \) and \( \beta = 4 \), we have obtained the uniform stationary state \( \phi_s = 0.85 \) in each finite element.

With \( \alpha > \beta \), the evolution of the density indicate that the process converges to the unstable state and confirmed also the analytical results. To begin with, we take \( \alpha = 3 \) and consider that the exponent parameter \( \beta \) was augmented (\( \beta \) ranges from \( \beta = 0.5 \) to \( \beta = 1.5 \)) and the initial density \( \phi_0 = 0.6 \). As shown in Figs. 2 and 3, the evolution of the density indicates that the process converges to a configuration with an irregular structure. Such a process is probably liable to chaotic behavior, may be described by a fractal, and is confirmed the stability study. Indeed, the structure becomes inhomogeneous and two struts are formed with transverse connectivity so that in some respects, these organizations do resemble the architecture of the trabecular bone. On the other hand, changing the range of \( \beta \) has an influence on the morphology. When \( \beta \)
increases, the connectivity is lowered and the struts are smaller. We show clearly that the value of $\alpha - \beta$ plays a major role in the prediction of a heterogeneous architecture. Similar results were also obtained for different values of the parameters $\alpha$ and $\beta$ (always for $\alpha > \beta$). Furthermore, similar numerical simulations in the unstable situation have been conducted to study the influence of the initial condition. Thus, we consider the same previous parameters with a different fixed initial density value $\phi_0$ ($\phi_0$ ranges from $\phi_0 = 0.6$ to $\phi_0 = 0.9$). We show in Figs. 4 and 5 the well-known result that a chaotic system is extremely sensitive to initial conditions [21]. It is striking that when $\phi_0$ increases, the thickness of the struts and their number increase. In the case $\phi_0 = 0.9$, struts appear near the external plate. Thus, the results show that the density mimics a cortex, with high density at the edges, surround-

The results presented here clearly show that this kind of model has the potential to predict reasonable trabecular morphology. We show that the value of certain parameters of this model plays a major role in the heterogeneous spatial organizations. The example simulated in this study and the resulting density distributions may be representative of some in vivo situations, such as compression in spinal segment.

References