A Study of Local Hydrodynamics in a 90° Branched Vessel with Extreme Pulsatile Flows

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A Study of Local Hydrodynamics in a 90° Branched Vessel with Extreme Pulsatile Flows

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To validate the pathological flow condition as the etiology of site-specific atherosclerosis, computer simulation and analysis are presented. Two extreme pulsatile flows, as have been measured in human femoral arteries, are used to numerically simulate in-vivo blood flow. The pulse with a steep temporal velocity gradient is assigned as "steep" pulse whereas the one with a lesser slope is called "less-steep" pulse. The effect of the extreme two pulses on the injury of endothelial cells in a 90° femoral artery of human is investigated by calculating flow parameters including instantaneous wall shear stresses. At the proximal and the distal branch apex, the oscillating wall shear stresses calculated from the 'steep' pulse cycle are found to be substantially greater than those from the 'less-steep' one. In contrast, for a straight artery, insignificant changes in flow parameters are observed for the extreme two pulses. It is evident from the calculated local wall shear stress that pathological changes can occur in the intima cells of the branch arterial wall and less likely in the straight arterial wall. An alteration of the pulsatile flow condition from a 'steep' to a 'less-steep' pulse, as may be achieved by pharmaceutical, chemical, or mechanical means, may be one of the possible ways to reduce the risk, thus slow the progression of or cause regression of arterial diseases, such as heart attack and stroke.

Keywords: Hydrodynamics, pulsatile flow, 90 degree branch, femoral artery, shear stress, pressure drop, pathological flow, etiology, atherosclerosis

1. INTRODUCTION

A hypothesis proposes that an amplified protosystolic pulse injures the arterial wall by first overstretching (thus inducing arteriosclerosis), and then by pathological level of shear stress on the intima (inducing atherosclerosis) (Kensey, K. R. and Cho, Y. I. [1994, 1992]). An amplified protosystolic pulse is defined as an injury-inducing blood flow caused by the left ventricle that is contracting too vigorously for a particular blood pressure and blood viscosity.

A wide range of blood flow velocities and time intervals of the flow cycle in human femoral artery
have been reported in the past (Risoe and Wille [1978]). They have found that the systolic rise time for a human femoral artery varies from 75 to 125 ms whereas backflow duration may have a range between 125–200 ms.

From the above hypothesis and velocity measurements it is evident that temporal slope of velocity of a femoral pulse may vary significantly which, in turn, may result in varied level of flow parameters at the arterial branch location from one individual to the other. Considering the fact mentioned before, the present study intends to quantify critical flow parameters in the branch region of an artery thereby identifying individuals whose pulse shape may be injury-inducing in nature.

The human circulatory system consists of many anatomically different arterial branches (Fig. 1A), with various branch angles, diameter ratios of the branch to the main lumen, downstream to upstream main lumen diameter ratios; and main lumen and branch curvatures. Since there is little in-vivo information on branch to main lumen flow rate ratio, m_b/m_a as a function of branch size relative to main lumen for other than major arterial bifurcations are available, a further investigation on the ratio m_b/m_a is needed (Cho [1988]). It is evident that space average flow ratio m_b/m_a is not only dependent on the size of the artery but strongly depends on the chosen instant of time during a pulse cycle. Also it may be noted that diameter and pulse shape may vary significantly (Back et al. [1987], Risoe and Wille [1978]). Hence, it is practically impossible to quantify a firm space average flow ratio m_b/m_a which keeps varying with time over a pulse cycle.

As seen in Figure 1B, lesions often occur at these branch sites (Nerem and Levesque [1987]) and subsequently, the fluid dynamics of arterial flow and the related mass transport have been implicated as factors responsible for the genesis of atherosclerotic lesions (Fry [1973]). Consequently, much work is needed to sort out the various geometrical and hydrodynamic variables that govern the fluid dynamic behavior in branch flow.

FIGURE 1  Segments of femoral artery angiogram of man is shown in Figure 1A. Vessel sizes are in mm. The striking symmetric locations of apparent lesions opposite branches are demonstrated in Figure 1B. Blood flow is from top to bottom.

Non-Newtonian fluid characteristics of blood in branch flow with pulsatility create complexities in carrying out a quantitative analysis. Flow impingement which occurs at a branch junction often produces a thinning of the shear layer in the main
vessel and branch downstream of the impingement location, resulting in a local increase in the shear stress. Flow separation is often observed in the branch site, but the conditions causing separation are not well documented [Lutz et al. [1983]. Liepsch et al. [1982] have measured and calculated flow parameters of a laminar flow in a 90° bifurcation. Increase in Reynolds number and branch flow ratio have caused an increase in the size of the recirculation zone in the main lumen wall opposite to the branch. Moravec and Liepsch [1983] have experimentally investigated effects of Newtonian and non-Newtonian fluids for a steady flow in a 3-D doubly branched renal artery and have found differences in flow profile distal to the bifurcations.

Detailed statistics from an earlier clinical trial studying the effects of lipid lowering on human atherosclerosis have been reported by Cho et al. [1985]. Angiograms from 147 patients have clearly indicated strong hydrodynamic effects on the development of atherosclerosis in femoral arterial vessels. Branch related lesions are classified as i) plaques along the opposite wall to the branch and ii) proximal and distal constrictions of the branch itself. In addition, the discrete lesions appear to be located along the inner curvature of the branch sites, and hardly any lesions are found along the outer curvature. Earlier experimental studies by Back et al. [1986] and Cho et al. [1985] have isolated effects of branch flow rate, branch angle, and the Reynolds number on pressure and flow fields in arterial branch models. However, these studies are limited to the region opposite to the branch only. The overall flow field and flow separation in the branch itself have not been reported.

Perktold [1989] have conducted non-Newtonian blood (Casson model) flow calculations in an arterial bifurcation. Velocity, wall shear stress, and non-Newtonian viscosity results are presented for a bi-phasic flow (a forward flow followed by a reverse flow) for a main lumen diameter of 4 mm and diameter ratio of 0.5. However, Perktold's study can not be related to physiological flow in the femoral artery of humans because the instantaneous flow considered is not tri-phasic (a forward flow, then a reverse flow followed by a forward flow) in nature and the diameter is smaller compared to the adult femoral arterial size of 6 to 7 mm. Khodadadi [1992] has reported wall pressure and shear stress variation along with the dynamic recirculation zone in a 90° branch geometry having rectangular cross-section, which however, is physiologically unrealistic. Due to the tri-phasic nature of the femoral pulse, milder recirculating flows are observed at the wall of the main lumen located near the branch and the recirculating zones are functions of both temporal and spatial parameters (Banerjee et al. [1993]). They have also found that during a cardiac cycle, enhanced fluctuation of shear rate and shear stress from positive to negative values are observed near the branch region.

The objective of the present study has been to numerically quantify and analyze temporal and spatial dependent flow parameters, including velocity profile, shear rate, shear stress and pressure drop in a branch junction arterial model in order to understand the effect of 'steep' and 'less-steep' pulse cycles on the flow field at the site-specific lesion formation regions.

2. METHODS

Geometry: The present investigation is based on a femoral artery angiogram (Fig. 1). The focus of this study is on a branch angle of 90°, which is commonly observed in the downstream portion of the femoral artery. The ratio of branch to main lumen width \(d_b/d_m\) is 0.4. The arterial branch model has upstream and downstream main lumen widths of 0.631 cm and branch width of 0.252 cm which is somewhat similar to femoral artery branches of human reported earlier by Cho et al. [1985].

Formulation: This study deals with the time-dependent solution of an incompressible, non-Newtonian fluid for the selected geometry. The flow is described by the conservation equations of fluid
mass and momentum. A finite element method (FEM) is used to solve the following two conservation equations to obtain the velocity, pressure, wall shear rate and shear stress distributions;

\[ u_{j,i} = 0 \]  

\[ \rho \left[ \frac{\partial u_i}{\partial t} + u_j u_{i,j} \right] = \sigma_{j,i} + \rho f_i \]  

where \( i, j = 1,2 \) for 2-D flow. \( u_i \) is the \( i \)th component of the velocity vector, \( \rho \) is density, \( \sigma_{ij} \) is stress tensor, and \( f_i \) is the body force. Further, \( \sigma_{ij} \)

\[ \sigma_{ij} = -\rho \delta_{ij} + \tau_{ij} \]  

where

\[ \tau_{ij} = 2 \eta_{ij} \epsilon_{ij} \]  

\[ \epsilon_{ij} = 0.5(u_{i,j} + u_{j,i}) \]  

Here, \( p \) is the pressure, \( \tau_{ij} \) is the deviatoric stress tensor, \( \epsilon_{ij} \) is the shear rate tensor, \( \eta_{ij} \) is the tensor viscosity, and \( \delta_{ij} \) is the Kronecker delta. Axial components of flow along the main lumen are reported along the \( x \) and \( y \) directions where \( y \) direction is parallel to the branch axis.

The stress vector \( s_i \) at a point on the boundary of a fluid element is defined by

\[ s_i = \sigma_{ij} n_j \]  

For a known element and the solution field, the stress component \( s_i \) on the boundary at the Gaussian integration points is evaluated. Subsequently, the normal and tangential components of stress vectors are obtained after applying appropriate transformations.

The Galerkin formulation (Baker [1983]) using nine nodal quadrilateral elements is applied herein to discretize the above continuity and momentum equations, which result in a set of nonlinear algebraic equations of the form

\[ M \frac{du}{dt} + K(u)u = F \]  

where \( K(u) \) is the global system matrix developed from the momentum balance, \( M \) is the mass matrix, \( u \) is the velocity unknown, and \( F \) is the forcing function (including body forces and boundary conditions).

The mesh plot for the artery is shown in Figure 2. Spatial variations of velocity across the flow, at various time steps, are obtained at axial locations marked by 1-s to 8-s for main lumen and 1-b to 8-b for branched lumen. Time histories of shear rate, shear stress and pressure drop are reported for both wall elements located at axial locations of 1-s to 8-s for main lumen and 1-b to 8-b for branched lumen. Spatial variations of shear stress, at various time steps, are obtained at axial wall locations for main and branch lumen. Time histories of wall shear stress at critical wall locations of both main and branched lumens are also presented here.

Often the FEM is not directly applied to the system of equations but rather to a perturbed system of equations in which the continuity requirement is weakened by

\[ u_{i,j} = -\epsilon p \quad \text{where} \quad \epsilon = 1 \times 10^{-9} \]  

For the studied geometry a small penalty parameter of the order of \( 1 \times 10^{-9} \) helps to conserve the mass balance within an accuracy of 0.05%. In other words smaller the penalty parameter, more stringent is the mass balance calculation. Also for high aspect ratios of the elements, a small penalty parameter is recommended (Banerjee et al. [1992]). Physically, this can be equated to simulating the flow having an insignificant compressibility effect. This approach has the advantage of temporarily eliminating one of the dependent variables, \( p^* \), which is later recovered by post-processing from the velocity field by

\[ p^* = -u^*_{i,j}/\epsilon \]
FIGURE 2 The mesh plot along with dimension of a 90° branch femoral artery. Also shown are the various axial locations (1-s to 8-s for main lumen and 1-b to 8-b for branch lumen) for which flow data are reported.

Clearly, both the pressure and velocity fields are determined in the calculation method, as must be the case.

The matrix equation (7), representing a discrete analog of the original equations for an individual fluid element, is constructed, assembled, and solved. For spatial integration, the number of iteration steps is limited to ten at each time step with a combination of the successive substitution method and quasi-Newton scheme. The numerical simulation of a pulsatile flow requires a time integration method. The implicit time integration scheme used in the current study is the second order trapezoidal method with a variable time step, which depends on the magnitude of temporal input velocity and its gradient change. Depending on temporal variation of velocity the time steps varied between $1 \times 10^{-3}$ to $1 \times 10^{-5}$ s. The finite element computer code FIDAP [1991] is used to formulate and solve this matrix equation. The IBM-3090 is used with TEMPLATE graphics for post-processing, and the results are down-loaded to an Apple-Macintosh computer.

In comparison to the core elements of the lumen the element sizes near the wall are kept smaller to achieve a better accuracy of flow parameters. The aspect ratio of nine-nodal quadrilateral elements is chosen to be less than 10. For validation of the numerical computation, two separate computer modeling runs at peak systolic flow with different convergence criteria have been performed. These runs are as follows: both the relative velocity error with respect to a previous step and the relative residue error as compared to the initial value are set to be 2% and 1% respectively (Banerjee et al. [1995, 1996]). Furthermore, the overall convergence is confirmed by increasing the total number of meshes by 20% from that of the previous run, and the two results are compared to check for accuracy. When the improvement with 20% more meshes is less than 1% in velocity vectors, wall shear stress, and pressure, the computation is considered accurate. The analysis of results is from the computation with the least CPU time, i.e., less than 2% for both relative velocity and relative residue errors.
Though this study is limited and does not explore complete 3-D flow phenomenon in a 90° branch artery, the intent of this 2-D flow simulation is to quantify and compare the changes between dominant components of flow parameters caused by 'steep' and 'less-steep' pulse cycles. As compared to a 2-D pulsed flow simulation, a 3-D pulsed flow requires large CPU time and memory, both of which are beyond the scope of the present computation facility.

**Boundary Conditions:** Since there isn't any available measured in-vivo spatial distribution of velocity data along the diameter of a femoral artery over an entire pulse cycle, it is extremely difficult to specify an accurate spatial and temporal dependent inflow boundary condition. In other words, though the time dependent core velocity is well documented (Risoe and Wille [1978]), the velocity distribution near wall region over the pulse cycle is unknown. Due to the non availability of complete inlet spatial velocity data, a parabolic or an uniform velocity profile at each instant of time may be considered. It is reasonable to assume that with these two types of spatial boundary conditions both extreme flow conditions or, in other words, two physiologically limiting cases can be studied (Banerjee [1992]).

The normalized instantaneous tri-phasic inlet velocity profile \( \left( \frac{u}{u_{peak}} \right) \) showing 'steep' and 'less-steep' pulses (Fig. 3) have been replotted from the in-vivo velocity profile measurement (Risoe and Wille [1978]) at the core region i.e. at the center of the artery. A 'steep' pulse is identified as an elevated protosystolic acceleration of blood flow for a specific blood pressure and blood viscosity.

Using an ultrasound Doppler flow meter, Risoe and Wille [1978] have measured instantaneous in-vivo velocity profiles for several human femoral arteries. Their measurement shows a wide range of pulse shape from one individual to another. It is considered that the 'steep' pulse is the one which has the highest velocity gradient during systole whereas the 'less-steep' one has the least velocity gradient. Also for the 'steep' pulse the time duration of the forward and reverse flows within a pulse cycle is less than the 'less-steep' one. For both velocity profiles, average flow rates over a period of pulse cycle is same.

In the present study, the centerline normalized tri-phasic inlet velocity profile is used as an instantaneous input for numerical calculations. Considering the relatively long entry length of the femoral artery, for each time step of the pulse cycle, a fully developed flow is used as a spatial inlet velocity distribution (Schultz [1972] and Dewey [1978]). In other words, at each time step, the peak velocity of the parabolic spatial inlet velocity changes according to the measured instantaneous centerline inlet velocity profile which is shown in Figure 3.

As shown in Figure 3 during the accelerating phase of the systole, the peak value of the parabolic flow, having positive value, increases with time whereas it decreases during the decelerating phase of the systole. In contrast, during early diastole the parabolic velocity is negative whereas during late diastole the velocity is again positive. For the 'steep' pulse, the input peak axial velocity \( (u_{peak}) \) is 36.3 cm/sec at \( t = 0.122 \) s, whereas for the 'less-steep' case it is 31.6 cm/sec at \( t = 0.139 \) s. The selected time steps for flow analysis have been marked from No. 1 to No. 8 in Figure 3. The inlet velocity component in y-direction is zero. The
no-slip boundary condition has been specified on the rigid arterial wall.

For the problem in hand, the stress-free boundary condition that arises naturally from the application of the finite element method to the flow equations is adequate at the outlet. The normal component of stress vector from equation 6 above can be written as

$$\sigma_n = \sigma_n n_i = -p + \mu (u_{ij} + u_{ji}) n_i n_j$$

(10)

The natural boundary condition forces $\sigma_n$ to be equal to zero (in a weighted sense). If, as in the present case, the viscous contribution to the normal stress is small, then the effect of this boundary condition is to force the pressure to be closed to zero at the outflow. Based on the flow resistance and stress-free outlet boundary condition, upstream pressures, including inlet pressure, are calculated in the program.

Non-Newtonian blood viscosity: In the present investigation, the Carreau model is used to represent the shear rate dependent non-Newtonian blood viscosity whose model constants are obtained by curve-fitting of available shear-rate dependent blood viscosity data in the literature (Cho and Kensey [1991])

$$\eta = \eta_{\infty} + (\eta_n - \eta_{\infty}) [1 + (\dot{\gamma}/\lambda)^n]^{\frac{n}{n-1}}$$

(11)

where $\gamma$ (characteristics time) = 3.313 s, $n = 0.3568$, $\eta_n = 0.56$ poise, $\eta_{\infty} = 0.0345$ poise.

In order to calculate viscosity in the flow field locally, the local shear rate, $\dot{\gamma}$, is calculated from velocity gradient through the second invariant of the rate of strain tensor, $II\dot{\gamma}$, as follows:

$$\dot{\gamma} = \sqrt{\frac{1}{2} II} = \sqrt{\frac{1}{2} \sum_{i,j} \dot{\gamma}_{ij} \dot{\gamma}_{ij}}$$

(12)

After the local viscosity is determined using Carreau model, Eq. (11), the local shear stress, $\tau (= \eta \dot{\gamma})$, is calculated. A factor of only 0.6% compliance is reported for a resting femoral artery (Nicholes and O'Rourkee [1990]) of human. Further, compliance of artery reduces significantly with age and at a later age atherosclerotic condition of artery, which is a rigid wall condition, may occur. Hence, a viscoelastic wall has been ignored for the present study. Also study of viscoelastic wall is beyond the scope of this study.

Blood with a constant density of 1.05 g/cm$^3$ was used. The pulse rate is 75 bpm. Hence, the dimensionless frequency parameter [$\alpha = 0.5 \frac{d}{(\omega/u)^{0.5}}$] for the femoral artery of man is found to be 4.87.

RESULTS AND DISCUSSION

This section contains an insight into the difference in various spatial and temporal distribution of flow parameters including velocity profile, shear rate, shear stress, and pressure drop in a 90° human femoral artery for 'steep' and 'less-steep' pulses that have parabolic spatial inlet velocity profiles. At the vicinity of reference location 1-s, for all times, the velocity profile is invariant along the axial direction. The reference point is chosen in such a way that there is no numerical instability in the wall pressure due to the entrance effect.

Earlier measurements by Back et al. [1987] showed that for a femoral branch artery the space average flow ratio $m_s/m_0$ can vary between 0.3 and 0.5. Hence, before performing a transient calculation, a computation at the peak systolic flow was performed in order to determine the main and branch artery length, keeping the pressure at the outlet to be zero, such that the average flow ratio $m_s/m_0$ is 0.3 which is a physiologically realistic number.

Further, at this point, it may be noted that the focus of this study is to investigate the effect of extreme physiological pulses on a femoral branch which is realistic in nature. Considering the computation time and space constraint for this publication and, of course, without deviating from the focus of this study, the authors have simplified the flow domain and the problem definition and have presented results which are important and relevant in explaining the results and conclusion.
Velocity Profile

Figures 4.1A–4.1D and 4.11–4.1L show axial velocity profiles at different locations (Fig. 2) along the main and branch lumen at desired time steps (Fig. 3) for the 'steep' pulse. At \( t = 0.044 \) s, Figures 4.1A and 4.1I show velocity profiles at axial locations in main lumen (1-s to 8-s) and branch lumen (1-b to 8-b), respectively. Velocity plots at time \( t = 0.044 \) s (Figs. 4.1A and 4.1I) and 0.097 s (Figs. 4.1B and 4.1J) represent accelerating flows in systole, whereas those at time \( t = 0.160 \) s (Figs. 4.1C and 4.1K) and 0.202s (Figs. 4.1D and 4.1L) represent decelerating flows in systole.

Figures 4.2A–4.2D and Figures 4.21–4.2L show axial velocity profiles at different locations along the main and branch lumen at the desired time steps for the 'less-steep' pulse. At \( t = 0.064 \) s, Figures 4.2A and 4.21 show velocity profiles at axial locations in main lumen (1-s to 8-s) and branch lumen (1-b to 8-b), respectively. Velocity plots at time \( t = 0.064 \) s (Figs. 4.2A and 4.2I) and 0.125 s (Figs. 4.2B and 4.2J) represent accelerating flow in systole whereas those at time \( t = 0.170 \) s (Figs. 4.2C and 4.2K) and 0.232 s (Figs. 4.2D and 4.2L) represent decelerating flows in systole. Due to limitation of space, velocity profiles during diastole for both 'steep' and 'less-steep' pulses are not reported here. Detail of velocity profiles during diastole may be obtained from the study by Banerjee, Cho, and Back [1993].

For the accelerating flow in systolic part of the 'steep' pulse, the proximal branch wall shows an increased negative flow (i.e., arrow #1 in Figs. 4.1I and 4.1J) from \( t = 0.044 \) s, where the axial velocity is 0.14 cm/s, to \( t = 0.097 \) s where the axial velocity is \(-1.96 \) cm/s. In comparison, for accelerating flow in systolic part of the 'less-steep' pulse, the proximal branch wall shows less negative flow (i.e., arrow #1 in 4.2J) at \( t = 0.125 \) s where the axial velocity is \(-1.71 \) cm/s. Although the main lumen flow is positive and accelerating, the negative flow near the proximal apex of the branch indicates the existence of an early recirculating region. However, the distal branch wall shows a sharp velocity gradient and higher positive velocity of 22.1 cm/s for plot No. 2-b (i.e., arrow #2 in Fig. 4.1J) for the 'steep' pulse. In contrast, for the 'less-steep' pulse, the distal branch wall shows a lower positive velocity of 20.9 cm/s for plot No. 2-b (i.e., arrow #2 in Fig. 4.2J).

During the decelerating phase of systole, at the proximal branch wall, three features: the recirculation intensity, measured by the magnitude of negative velocity; the recirculation length, measured by the axial distance of negative velocity; and the recirculation height, measured by the lateral extent of negative velocity. For plot No. 1-b the magnitude of the maximum negative velocity is \(-1.57 \) cm/s (Fig. 4.1K) for the 'steep' pulse whereas it is \(-1.33 \) cm/s (Fig. 4.2K) for the 'less-steep' pulse. Similarly, for plot No. 1-b the maximum negative velocity is \(-6.53 \) cm/s (Fig. 4.1L) for the 'steep' pulse whereas it is \(-5.15 \) cm/s (Fig. 4.2L) for the 'less-steep' pulse, which indicates a higher intensity of recirculation for the 'steep' pulse.

For the accelerating flow in systolic part of the 'steep' pulse, the lateral extent of negative velocity is 0.078 cm from the proximal wall whereas for plot No. 1-b (Fig. 4.1L) it is 0.17 cm. When compared with the 'steep' pulse, for plot No. 1-b (Fig. 4.2K) of the 'less-steep' pulse, the lateral extent of negative velocity is 0.061 cm from the proximal wall whereas for plot No. 1-b (Fig. 4.2L) it is 0.16 cm, which indicates a comparatively lower recirculation height at the late deceleration phase of systole. The axial extent of negative velocity is observed until plot No. 6-b (Fig. 4.1K) for the 'steep' pulse as compared to plot No. 5-b (Fig. 4.2K) for the 'less-steep' pulse. In Figures 4.1L and 4.2L the axial extent is prolonged further downstream, which indicates an
FIGURE 4  Comparison of velocity distributions across the main and branch lumens at different axial locations for various time steps of 'steep' (Fig. 4.1A-4.1D and 4.1L) and 'less-steep' (Fig. 4.2A-4.2D and 4.2L) pulses.
FIGURE 4 (Continued).
increasing recirculation length at the late deceleration phase of systole. In addition, for the decelerating phase of systole, the distal branch wall shows a larger decrease in positive velocity for the 'less-steep' pulse (i.e., arrow #2 for plot No. 1-b ~ 5-b in Fig. 4.1K and 1-b ~ 3-b in Fig. 4.1L) as compared to the 'steep' pulse (i.e., arrow #2 for plot No. 1-b ~ 5-b in Fig. 4.2K and 1-b ~ 3-b in Fig. 4.2L). A generation of recirculating flow in the distal wall of the branch lumen is observed during early systole. In addition, the co-existence of a high velocity at the distal branch wall and a low velocity at the proximal branch wall is observed.

Interestingly, during the decelerating phase of systole, another recirculating region develops at the main lumen wall opposite to the branch. Formation of recirculating flow is observed at \( t = 0.160 \) s (i.e., arrow #1 for plot No. 7-s ~ 1-s in Fig. 4.1C) for the 'steep' pulse. In contrast, formation of a milder recirculating flow is observed at \( t = 0.170 \) s (i.e., arrow #1 for plot No. 7-s ~ 1-s in Fig. 4.2C) for the 'less-steep' pulse. Subsequently, the intensity, length, and height of the recirculation increase as the flow reaches late systole at \( t = 0.202 \) s (i.e., arrow #1 for plot No. 7-s ~ 1-s in Fig. 4.1D) for the 'steep' pulse and at \( t = 0.232 \) s (i.e., arrow #1 for plot No. 7-s ~ 1-s in Fig. 4.2D) for the 'less-steep' pulse. Further, at the top wall of the main lumen, near the branch apex, mild recirculations are observed (i.e., arrow #2 for plot No. 2-s ~ 1-s and arrow #3 for plot No. 7-s ~ 4-s in Figure 4.1D for the 'steep' pulse and also in Figure 4.2D the 'less-steep' pulse).

As reported by Banerjee [1992], during early diastole, the recirculating zone at the main lumen wall is further reinforced by a negative inlet flow and also a complete flow reversal in the branch region is observed, indicating a suction of fluid from the proximal branch wall to the main lumen. Velocity profiles in mid-diastole are similar to the systolic flow, but with lower magnitudes.

Shear Rate

Shear rate calculations for the 'steep' pulse are shown in Figures 5.1A - 5.1D, whereas Figures 5.2A - 5.2D show shear rate calculations for the 'less-steep' pulse at different axial locations of the wall region. For wall definitions, Figure 3 may be referred to. The main lumen opposite to the branch, denoted as the bottom wall, shows a high shear region at \( t = 0.092 \) s for the 'steep' pulse and at \( t = 0.095 \) s for the 'less-steep' pulse, which represents a high instantaneous velocity gradient during systole. For the 'steep' pulse the maximum shear rate near the inlet section decreases from \( \gamma = 310 \) s\(^{-1} \) (i.e., arrow #1 for plot No. 1 in Fig. 5.1A) to a reduced positive value of \( 216 \) s\(^{-1} \) at the branch region (i.e., plot No. 5 in Fig. 5.1A). In case of the 'less-steep' pulse the maximum shear rate at the same location decreases from \( \gamma = 240 \) s\(^{-1} \) (i.e., arrow #1 for plot No. 1 in Fig. 5.2A) to a reduced positive value of \( 173 \) s\(^{-1} \) (i.e., plot No. 5 in Fig. 5.2A). As discussed earlier, a prolonged recirculating region is observed at the bottom wall from \( t = 0.152 \) s (i.e., systolic decelerating flow) to \( t = 0.32 \) s (i.e., mid-diastole) for the 'steep' pulse. Similar recirculating region is observed at the bottom wall from \( t = 0.193 \) s (i.e., early decelerating systole) to \( t = 0.36 \) s for the 'less-steep' pulse. The maximum negative shear rate is \( -274 \) s\(^{-1} \) (i.e., arrow #2 plot No. 1 in Fig. 5.1A) for the 'steep' pulse whereas it is \( -221 \) s\(^{-1} \) (i.e., arrow #2 plot No. 1 in Fig. 5.2A) for the 'less-steep' pulse.

It is interesting to note that the top wall of the main lumen near the branch apex shows a very high shear rate. Although the top wall in Figure 5.1B and 5.2B shows a trend for shear rate similar to that in Fig. 5.1A and 5.2A, the apexes of the proximal branch wall (i.e., arrow #2 for plot No. 5 in Fig. 5.1B and 5.2B) and the distal branch wall (i.e., arrow #2 for plot No. 5 in Fig. 5.1B and 5.2B) show elevated positive shear rates. In case of the 'steep' pulse, for plot No. 2 in Figure 5.1B, the shear rate increases threefold to a value of \( 907 \) s\(^{-1} \) at \( t = 0.104 \) s, whereas the shear rate for plot No. 5 increases to a value of \( 711 \) s\(^{-1} \) at \( t = 0.117 \) s. For plot No. 2 in Figure 5.2B, the shear rate for the 'less-steep' pulse increases to a value of \( 709 \) s\(^{-1} \) at \( t = 0.122 \) s, whereas the shear rate for plot No. 5 increases to a value of \( 565 \) s\(^{-1} \) at \( t = 0.142 \) s.
FIGURE 5 Comparison of time histories of wall shear rate distributions at various wall locations in main and branch lumens for 'steep' (Fig. 5.1A–5.1D) and 'less-steep' (Fig. 5.2A–5.2D) pulses.
The proximal wall near the branch apex (Fig. 5.1C) experiences a negative shear for a prolonged length of time, i.e., from \( t = 0.05 \) s to \( t = 0.34 \) s (i.e., arrow #1 for plot No. 1 in Fig. 5.1C) for the 'steep' pulse and \( t = 0.058 \) s to \( t = 0.392 \) s (i.e., arrow #1 for plot No. 1 in Fig. 5.2C) for the 'less-steep' pulse. A maximum negative shear rate of \(-514 \) s\(^{-1}\) at \( t = 0.243 \) s (i.e., arrow #2 for plot No. 1 in Fig. 5.1C) is observed for the 'steep' pulse whereas it is \(-269 \) s\(^{-1}\) at \( t = 0.273 \) s (i.e., arrow #2 for plot No. 1 in Fig. 5.2C) for the 'less-steep' pulse. At a further downstream location of the proximal branch wall, the intensity and duration of the recirculating flow significantly decrease with time. During systole, downstream locations of the proximal branch wall experience the maximum value for shear rate at different times (i.e., slant arrow #3 for plots No. 1 to 8 in Fig. 5.1C and 5.2C), indicating that the recirculation oscillates with time and in space. As expected, the distal branch wall adjacent to the distal branch apex shows very high positive shear rates. During systole, a shear rate of \( 1240 \) s\(^{-1}\) is observed at \( t = 0.11 \) s (i.e., arrow #1 plot No. 1 in Fig. 5.1D) for the 'steep' pulse whereas a shear rate of \( 998 \) s\(^{-1}\) is observed at \( t = 0.142 \) s (i.e., arrow #1 plot No. 1 in Fig. 5.2D) for the 'less-steep' pulse. The higher shear rate is localized in nature and decreases as the fluid reaches the downstream location of the branch lumen.

Shear Stress

The wall shear stress, which shows a combined effect of the non-Newtonian viscosity and shear rate, is presented in Figures 6.1A–6.1D for the 'steep' pulse and Figures 6.2A–6.2D for the 'less-steep' pulse. At the downstream location of both main and branch lumens the shear stress variation is oscillatory in nature (i.e., arrow #2 in Fig. 6.1A), due to the tri-phasic nature of the pulse, having values in the range of 16 dynes/cm\(^2\) to \(-12 \) dynes/cm\(^2\) for the 'steep' pulse and 13 dynes/cm\(^2\) to \(-8.5 \) dynes/cm\(^2\) (i.e., arrow #2 in Fig. 6.2B) for the 'less-steep' pulse. The oscillatory shear stress values vary as the fluid encounters the branch lumen. At \( t = 0.16 \) s, i.e., during the decelerating phase of systole for the 'steep' pulse, both negative and positive shear stress levels are observed (i.e., arrow #1 for plot No. 3 in Fig. 6.1A), indicating the existence of a recirculating region at that instant in time. In contrast, at \( t = 0.17 \) s, i.e., during the decelerating phase of systole for the 'less-steep' pulse, only a positive shear stress is observed (i.e., arrow #1 for plot No. 3 in Fig. 6.2B).

At the top wall near the proximal branch orifice, the shear stress for the 'less-steep' pulse reaches a magnitude of \( 40 \) dynes/cm\(^2\) at \( t = 0.125 \) s (i.e., arrow #1 for plot No. 2 in Fig. 6.2B) whereas it reaches \(-5.2\) dynes/cm\(^2\) at \( t = 0.232 \) s. In case of the 'steep' pulse the shear stress reaches a value of \( 48 \) dynes/cm\(^2\) at \( t = 0.097 \) s (i.e., arrow #1 plot No. 2 in Fig. 6.1B) whereas it reaches \(-6.7\) dynes/cm\(^2\) at \( t = 0.202 \) s, showing an increase in the magnitude of shear stress. At the top wall near the distal branch orifice, the shear stress for the 'steep' pulse reaches a value of \( 28.5 \) dynes/cm\(^2\) at \( t = 0.097 \) s (i.e., arrow #1 for plot No. 2 in Fig. 6.1B) whereas it reaches \(-6.2\) dynes/cm\(^2\) at \( t = 0.243 \) s. At the same location the shear stress for the 'less-steep' pulse reaches a comparatively lower magnitude of \( 27 \) dynes/cm\(^2\) at \( t = 0.125 \) s (i.e., arrow #2 for plot No. 2 in Fig. 6.2B) whereas it reaches \(-6.2\) dynes/cm\(^2\) at \( t = 0.296 \) s. The shear stress for the proximal branch wall near the entry region of branch shows a value of \(-33.6\) dynes/cm\(^2\) at \( t = 0.243 \) s (i.e., arrow #1 for plot No. 5 Fig. 6.1C) for the 'steep' pulse indicating a shift in oscillatory shear in a negative direction whereas the distal branch wall shows a value of \( 45.5 \) dynes/cm\(^2\) (i.e., arrow #1 for plot No. 2 in Fig. 6.1D), a shift in oscillatory shear in a positive direction. The shear stress for the 'less-steep' pulse at the same location becomes \(-24.2\) dynes/cm\(^2\) at \( t = 0.296 \) s (i.e., arrow #1 for plot No. 5 in Fig. 6.2C), indicating a comparatively reduced shift in the oscillatory shear. Also, at the distal wall a comparatively reduced shear stress of \( 44.1 \) dynes/cm\(^2\) (i.e., arrow #1 for plot No. 2 in Fig. 6.2D) is calculated for the 'less-steep' pulse.
FIGURE 6  Comparison of wall shear stress distributions along the wall of the main and branch lumen at different time steps of injurious (Fig. 6.1A–6.1D) and ‘less-steep’ (Fig. 6.2A–6.2D) pulses.
Figures 7.1A–7.1B show time history shear stress plots for the 'steep' pulse whereas Figures 7.2A–7.2B show for the 'less-steep' pulse. For the 'steep' pulse, at $t = 0.104$ s, the maximum wall shear stress of 50.9 dynes/cm$^2$ occurs at the top wall of the main lumen located near the proximal branch apex ($x(t) = 1.5$ cm in Fig. 7.1A). In comparison, for the 'less-steep' pulse, at $t = 0.122$ s, the maximum shear stress of 40.3 dynes/cm$^2$ occurs at the same location (Fig. 7.2A) indicating a 21% reduction in peak shear stress value. For both type of pulses the bottom wall, opposite to the branch ($x = 1.63$ cm in Figs. 7.1A and 7.2A), experiences a 33% reduction in peak shear stress as compared to the normal value ($x = 0.75$ cm in Fig. 7.1A and 7.2A). In the main lumen a 17% reduction in the peak negative shear stress value is observed for the 'less-steep' pulse ($-12.7$ dynes/cm$^2$ at $t = 0.273$ s) as compared to the 'steep' pulse ($-15.3$ dynes/cm$^2$ at $t = 0.236$ s).

For the 'steep' pulse, at $t = 0.117$ s, the maximum wall shear stress of 60.9 dynes/cm$^2$ occurs at the distal branch wall apex ($y = 0.63$ cm in Fig. 7.1B). In comparison, for the 'less-steep' pulse, at $t = 0.142$ s, the maximum shear stress of 49.9 dynes/cm$^2$ occurs at the same location (Fig. 7.2B) indicating a 18% reduction in peak shear stress value. In the branch lumen, a 23% reduction in the peak negative shear stress value at the proximal wall is observed for the 'less-steep' pulse ($-27.1$ dynes/cm$^2$ at $t = 0.273$ s) as compared to the 'steep' pulse ($-35.2$dynes/cm$^2$ at $t = 0.241$ s).

### Pressure Drop

The pulsatile pressure drop, $\Delta p = P_{p} - P_{s}$, for a 'steep' pulse is presented in Figures 8.1A–8.1D for the 'steep' pulse and Figures 8.2A–8.2D for the 'less-steep' pulse. The reference instantaneous
FIGURE 8  Comparison of time histories of pressure drop distributions for a pulse cycle at various wall locations in the main and branch lumen for injurious' (Fig. 8.1A–8.1D) and 'less-steep' (Fig. 8.2A–8.2D) pulses.
pressure is considered at wall locations of \( t \). Each plot represents the pressure drop at each axial location. A discrete bilinear pressure formulation is used to calculate the pressure data. The pressure drop becomes tri-phasic due to the tri-phasic nature of the input velocity curve. Significant pressure changes include an initial pressure drop and a subsequent pressure rise, followed by another pressure drop. For the 'steep' pulse the maximum pressure drop of \(-0.9 \text{ mm Hg}\) occurs at \( t = 0.055 \text{ s} \) whereas the maximum pressures rise of \(1.2 \text{ mm Hg}\) occurs at \(0.236 \text{ s}\). For the 'less-steep' pulse the maximum pressure drop of \(-0.71 \text{ mm Hg}\) occurs at \( t = 0.058 \text{ s} \) whereas the maximum pressure rise of \(0.8 \text{ mm Hg}\) occurs at \(0.142 \text{ s}\).

When the instantaneous times for the peak values of input velocity and pressure drop are compared, a phase lag is observed. For the accelerating flow of systole, the pressure drop increases with time from \( t = 0.0 \text{ s} \) to \( t = 0.055 \text{ s} \) for the 'steep' pulse and from \( t = 0.0 \text{ s} \) to \( t = 0.058 \text{ s} \) for the 'less-steep' pulse. In contrast, for the decelerating flow of systole and the accelerating negative flow of diastole, the pressure rises with time, until \( t = 0.236 \text{ s} \) for the 'steep' pulse and until \( t = 0.273 \text{ s} \) for the 'less-steep' pulse. As the flow accelerates in mid-diastole, the pressure again drops to almost zero at \( t = 0.332 \text{ s} \) for the 'steep' pulse and at \( t = 0.431 \text{ s} \) for the 'less-steep' pulse. The branch wall region contributes to the rise in the local pressure (shown by the + sign in Figs. 8.1A–8.1D and Figs. 8.2A–8.2D) a phenomenon which indicates a larger adverse pressure gradient. Since it is difficult to isolate regions of flow separation, a detailed knowledge of velocity, shear rate and shear stress is required, as demonstrated earlier.

From the above analyses it is evident that the 'less-steep' pulse generates a hydrodynamic condition for the artery that is more favourable than that generated by a 'steep' pulse. A comparison of 'less-steep' and 'steep' pulses in the branch region reveals that a lower magnitude and less shift in oscillatory shear rate and shear stress are obtained for the 'less-steep' pulse. Also, the recirculation length, height, and intensity are smaller for the 'less-steep' pulse. In addition, the levels of pressure drop and pressure rise are reduced for the 'less-steep' pulse.

**DISCUSSION**

In this section various physiological phenomena near a 90° branched artery are discussed. Some of the observations has already been reported by previous studies whereas other observations are due to combined effect of site-specific pulse shape and geometric configuration of the 90° branch artery.

It may be noted that only one cycle of the inlet velocity time variation is required to achieve a periodic flow. The flow parameters for each pulse cycle is independent of the other since there exists a dormant inflow condition, i.e., no flow during the late diastole (Fig. 3). Though the phase difference keeps increasing with time while the flow develops over a pulse cycle, during late diastole the velocity and pressure are in phase with each other. This is particularly true for the femoral pulse studied here, as it has a long duration of no flow condition during late diastole.

i) For a cardiac cycle, during systole, there are formation of two dominant recirculating zones e.g., one generated at the proximal wall of the branch lumen, and the other at the main lumen wall opposite to the branch. Further, due to the tri-phasic nature of the pulse, two additional recirculating zones are generated at the proximal wall and the wall opposite to the branch during mid-diastole. In comparison to mid-diastole, the recirculating zones during systole have a higher intensity, length and width. Also at the top wall, near the branch apex, mild recirculation zones are observed.

ii) The recirculating zones at branch locations not only oscillate with time but also move back and forth along the axial directions. In addition, recirculation height changes along the branch. Hence, the recirculating zones are functions of both temporal and spatial parameters.
iii) The changes in magnitude and direction of oscillatory wall shear stress values, adjacent to the entry region of branch lumen, are significantly higher than the straight lumen of the artery. For the 'steep' pulse, near the entry region of the branch, the distal wall shows higher shear stress with a value of 48 dynes/cm² as compared to a lower shear stress value of -33 dynes/cm² at the proximal wall. The main lumen wall near the proximal branch apex shows an oscillatory shear stress within the range of 48 dynes/cm² to -6.7 dynes/cm² which indicates a shift in positive direction as compared to the normal oscillatory wall shear stress. Also the main lumen wall near the distal branch apex shows a similar shift in the oscillatory shear stress within the range of 28 dynes/cm² to -6.2 dynes/cm². The main lumen wall, opposite to the branch, shows a larger range of oscillatory wall shear stress.

iv) The pressure drop becomes tri-phasic in nature due to the tri-phasic nature of the input velocity curve. Pressure changes include an initial pressure drop and subsequent pressure rise, followed by another pressure drop. The branch wall region contributes to the rise in the local pressure which indicates a larger adverse pressure gradient.

v) One of the inherent characteristic of a pulse flow is the phase lag between the velocity and the pressure (Banerjee et al. [1995]). The phase difference between velocity and pressure is relatively insignificant during the accelerating phase of systole but it keeps increasing during the decelerating phase of the systole and early diastole.

As reported in the earlier studies (Banerjee et al. [1993]), the time dependent velocity gradient, particularly the accelerating phase of the systole, is critical in determining the magnitude of critical flow parameters e.g., wall shear stress. Clearly, the focus of this study was the effect of different acceleration rate during systolic flow on a specified geometry. The study of the diastolic part is for academic interest only and it has insignificant effect on the outcome.

SUMMARY AND CONCLUSION

The dynamic recirculating vortex near the branch resulted in an oscillatory wall shear stress and a shear rate that depended both on time and space. Sharp changes in the magnitude and the direction of the oscillatory wall shear stress values, as compared to a normal value, created unfavorable physiological conditions that could lead to increasing damage to the arterial walls.

The intima layer of an artery can only function normally under an optimum range of shear value (Gibbons, G. H. and Dzau, V. J. [1994]). Adverse physiological conditions can either be due to a higher or a lower level of wall shear stress when it is compared with a normal range. A lower level of shear stress can cause lipid deposition and depletion of oxygen transport whereas mechanical damage to the endothelial cells may be due to higher levels of wall shear stress. Depending on the nature of pulse shape and blood viscosity, specially near the branch region, pathological flow conditions, i.e. higher or a lower level of wall shear stress may exist.

Following distinctions were observed for 'steep' and 'less-steep' pulses:

i) A 'less-steep' pulse generated a more favourable hydrodynamic condition for the artery than a 'steep' pulse.

(ii) A comparison between 'less-steep' and 'steep' pulses in a 90° branch region revealed that a lower magnitude and a shift in the oscillatory shear rate and shear stress were obtained for the 'less-steep' pulse.

To summarize briefly, adverse hydrodynamic conditions are observed for the 'steep' pulse. The 'steep' pulse, in certain locations, injures the intima layer of the arterial wall due to higher oscillating shear stress whereas in other locations a lower level of shear stress creates lipid deposition and retarded oxygen transport. The amount of damage can be lessened by the 'less-steep' pulse.

The process of atherosclerosis has been attributed to several different factors or combination
... of factors which include genetic reasons related to aging, bio-chemical and environmental factors. Since there is no consensus regarding the cause and effect of atherosclerosis, the authors have limited their study and presented results only from the hydrodynamic point of view. An alteration of the pulsatile flow condition from a 'steep' to a 'less-steep' pulse, as may be achieved by pharmacetical, chemical, or mechanical means, may be one of the possible ways to reduce the risk thus slow the progression of or cause regression of arterial diseases, such as heart attack and stroke.

**NOMENCLATURE**

\[ d \] = lumen width  
\[ m \] = mass flow rate  
\[ n \] = normal vector  
\[ p \] = pressure  
\[ P_{1-s} \] = wall pressure at reference location (initial node)  
\[ P_i \] = wall pressure at any downstream nodes  
\[ s_i \] = stress vector  
\[ t \] = time  
\[ T \] = period of heart beat  
\[ u_i \] = velocity vector  
\[ x \] = distance along the main lumen  
\[ y \] = distance along the branch lumen  
\[ z \] = dimensionless frequency parameter, or Womersley number  
\[ \dot{\gamma} \] = shear rate  
\[ \epsilon \] = penalty parameter  
\[ \eta \] = viscosity  
\[ \nu \] = kinematic viscosity (= \( \eta/\rho \))  
\[ \rho \] = density  
\[ \sigma_{ij} \] = shear stress  
\[ \omega \] = circular frequency, \( 2\pi/T \)  
\[ \tau_w \] = wall shear stress

**Subscripts**

\[ s \] = main lumen  
\[ b \] = branch lumen  
\[ \Delta \] = numerically computed valued  
\[ \ast \] = difference

**References**


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ology of arterial occlusive disease”, *Journal of Invasive Cardiology*, 6(2), 55–70.