PRESSURE-FLOW CHARACTERISTICS DURING PERISTALTIC TRANSPORT OF BINGHAM FLUID IN DISTENSIBLE TUBE WITH DIFFERENT WAVE FORMS

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ABSTRACT

The present study investigates the peristaltic transport of non-Newtonian Bingham fluid in a diverging tube with different wall wave forms: sinusoidal, multi sinusoidal, triangular, trapezoidal and square waves. Fourier series is employed to get the expressions for temporal and spatial dependent wall shapes for different wave forms. Solutions for time average pressure rise – flow rate relationship, $\overline{\Delta P_L}$-$\overline{Q}$, are computed for different amplitude ratios, φ, and yield stresses, τ_0. The results indicate that φ and τ_0 play a vital role in peristalsis. When φ of the sinusoidal wave is increased from 0.5 to 0.7, maximum pressure rise, $\Delta P_{L\,\max}$, for a Newtonian fluid, $\tau_0 = 0$ increased by a factor of 5. Increasing τ_0 from 0 to 1 for $\varphi = 0.5$ causes the $\Delta P_{L,\max}$ to decrease by ~25%. The $\overline{\Delta P}_{L,critical}$ is maximum for the square wave and minimum for the triangular wave (4 to 15 times less depending on φ). Thus, the results clearly invalidate the assumption of the fluid as Newtonian and demonstrate that a maximum flow rate can be achieved when the wall movement follows a square wave pattern.

INTRODUCTION

Research on the peristaltic transport is motivated by interest in the physiological fluid transport in many biological systems, e.g. urine transport from kidney to the bladder through the ureter, movement of chime, and transport of spermatozoa in the vas deferens. The initial study conducted by Jaffrin and Shapiro (1971) introduced the lubrication theory model in which the effects of fluid inertia and wall curvature are neglected. Eytan and Elad (2001) extended the same theory for a two dimensional channel and discussed in detail the phenomenon of trapping and reflux during the intrauterine fluid flow.

Most of the previous studies have been carried out by assuming the fluid to be Newtonian and the waveform to be sinusoidal. A food bolus passing through the esophagus or the movement of spermatozoa in the ductus efferentes may be suitably modeled by considering a non-Newtonian fluid. A limited number of literatures (Hariharan and Banerjee (2003)) are available for non-Newtonian fluid analysis of the peristaltic flow. In the present research, an attempt is made to study the peristaltic transport in diverging tubes with different wave forms, yield stress values, and amplitude ratios with specific application to the flow of spermatic fluid in the vas-deferens of a rhesus monkey.

PROBLEM FORMULATION

Governing Equations and Boundary Conditions

By applying the long wavelength approximation and neglecting inertia terms, the appropriate equation of motion after introducing non-dimensional variables reduces to

$$\frac{\partial p}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\left\{\left(\frac{\partial u}{\partial r}\right) + \tau_0\right\}\right];\ \frac{\partial p}{\partial r} = 0 \tag{1}$$

where p is the pressure, r is the radial distance, x is the axial distance, u is the axial velocity.

The boundary conditions are:

$$\partial u/\partial r = 0 \quad \text{at} \quad r = 0 \tag{2}$$

$$u = 0 \quad \text{at} \quad r = h = 1 + (\lambda kx/a_0) + \varphi(WaveForm) \tag{3}$$

where h is the wall co-ordinate, t is the time, and φ is the amplitude ratio.

From the Fourier series, the wave equation for sinusoidal wave form is obtained as

$$h = 1 + (\lambda kx/a_0) + \Phi \sin 2\pi(x - t) \tag{4}$$

An expression for axial velocity is obtained by solving the governing equation for u and applying the boundary conditions as

$$u(x,r,t) = (1/4)(dp/dx)(r^2 - h^2) - \tau_0(h - r) \tag{5}$$

The instantaneous volume flow rate in laboratory frame of reference is given by

$$Q(x,t)=\frac{h^4\pi}{8}(-dp/dx)(1-((8\tau_0/3h)/(-dp/dx))+((16\tau_0{}^4/3h^4)/(-dp/dx)^2)) \quad (6)$$

where Q is the volume rate of flow.

Integrating the above equation over the length of the tube, we get

$$Z=(-dp/dx)=\left[\frac{8Q(x,t)}{\pi}+(8\frac{\tau_0}{3})h^3\right]\Big/h^4 \quad (7)$$

This integral is solved numerically using the trapezoidal rule

RESULTS AND DISCUSSION

Figure 1 shows the velocity profile for a Bingham fluid. The solid plug zone is evident in figure 1 where the applied shear stress is below the yield stress, τ_0. Beyond this plug zone the shear stress is more than the τ_0 and the fluid behaves like a Newtonian fluid. Velocity of the semen at the tube entrance for sinusoidal wave is 0.035 m/s and decreases by ~7% as the τ_0 is increased from 0 to 1.

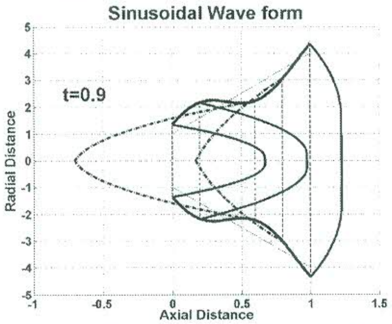

Figure 1: Velocity profile along the length of the distensible tube at t=0.9, φ=0.5, τ_0=1 for sinusoidal wave form

The pressure rise, ΔP_L, is plotted as a function of time, t, in figure 2 for φ = 0.5, 0.6, and 0.7. Due to space constraints, plots for the sinusoidal wave alone are shown in all the figures. For sinusoidal wave with τ_0= 0 (Newtonian fluid), as φ is increased from 0.5 to 0.6, the $\Delta P_{L,\max}$ increases from 5.8 to 13.3, which corresponds to a ~128% rise in $\Delta P_{L,\max}$ value. The $\Delta P_{L,\max}$ value rises by ~170% from 13.3 to 35.9, when φ is increased from 0.6 to 0.7. Similar results are obtained for all the other wave forms.

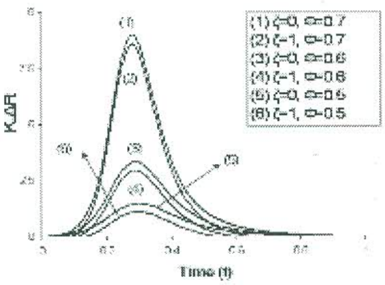

Figure 2: Temporal variations of pressure rise along the tube length.

Figure 2 shows that, $\Delta P_{L,\max}$ decreases with increase in τ_0. For φ = 0.5, increasing τ_0 from 0 to 1 causes the $\Delta P_{L,\max}$ to decrease by ~25% , ~15%, and ~15% for sinusoidal, trapezoidal, and square waves, respectively. However, for a higher φ value of 0.7, $\Delta P_{L,\max}$ drops only by ~5%, ~2.5%, and ~1.5% for the above mentioned wave forms. Thus, for higher degree of occlusion i.e large φ value, $\Delta P_{L,\max}$ produced by trapezoidal and square wave is not significantly changed by the yield stress, τ_0, of the fluid. However, the same conclusion cannot be drawn in the case of the sinusoidal wave where $\Delta P_{L,\max}$ changes significantly with τ_0 as seen in figure 2.

Figure 3 shows the time average pressure rise, $\overline{\Delta P_L}$, as a function of $\overline{Q}$. For a sinusoidal wave with φ = 0.5, $\overline{\Delta P}_{L,critical}$ ($\overline{\Delta P_L}$ for which the average flow rate is zero) decreases by ~83% from 1.5 to 0.3 when τ_0 is increased from 0 to 1. On the contrary, for a square wave with same φ, $\overline{\Delta P}_{L,critical}$ decreases only by ~23% for the same

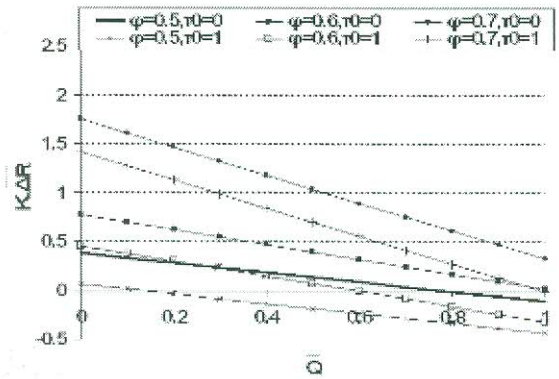

Figure 3: Dependence of average pressure rise - average flow rate relationship on φ and τ_0

rise in τ_0 value.

A maximum $\overline{\Delta P}_{L,critical}$ value of 27.3 is obtained for a square wave with φ = 0.7 and a minimum $\overline{\Delta P}_{L,critical}$ value of 0.02 is obtained for a triangular wave with φ = 0.5. It is observed that, for trapezoidal and square waves, the non-Newtonian effects (τ_0) of a Bingham fluid has lesser influence on the average pressure rise

CONCLUDING REMARKS

Results suggest that the $\overline{\Delta P_L}$ value has increased dependence on φ and τ_0. It has been shown that non-Newtonian fluids are more prone to reverse flow than Newtonian fluids. It would be further interesting to investigate the combined effect of yield stress and the power law index on the flow properties by employing the Herschel-Bulkley model for non-Newtonian fluid.

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