Proceedings of IMECE'04:
ASME International Mechanical Engineering Congress & RD&D EXPO
Anaheim, California, November 13-19, 2004

IMECE2004-59045

COUPLED OXYGEN TRANSPORT IN THE AVASCULAR REGION OF A CORONARY ARTERY FOR BASAL TO HYPEREMIC FLOW

Vinayak Vaidya[1] Lloyd H. Back[3] Rupak K. Banerjee[2]

(1) Mechanical Engineering Department,
(2) Biomedical Engineering Department,
University of Cincinnati, Cincinnati, OH.
(3)Jet Propulsion Laboratory, California Institute of Technology,
Pasadena, CA.

ABSTRACT

The numerical investigation of coupled oxygen transport to the avascular region of the wall of coronary artery is carried out for varying wall thickness and flow rates from basal to hyperemic condition. The factors affecting the O_2 transport, such as, consumption of oxygen in the avascular wall region, w, the avascular thickness, δ, supply of O_2 from *vasa vasorum*, nonlinear O_2 binding capacity of the hemoglobin and varying flow rates, are taken into account. The O_2 concentration boundary layer, δ_b, is observed to be of ~80μm thickness. The lowest medial partial pressure, $Po_{2,min}$, decreases by ~80% for a larger avascular thickness, δ, of 300 μm when compared with that for smaller δ of 200 μm.

INTRODUCTION

O_2 is supplied by radial diffusion from the vessel lumen to arteries of all species, and supplies from other sources (adventitial vessels and vasa vasorum) vary according to species and size of vessels [Crawford and Blankenhorn, 1991]. Microcathode measurements by Zemplényi et. al. [1989] in the iliofemoral arteries of rabbits with subintimal thickening, indicated the $Po_{2,min}$ to be 15 mmHg due to the proliferation of newly formed nutrient vessels in the adventitia that supplied O_2 to the outer part of the avascular wall. This adaptation is an important mechanism against hypoxia, and may be an essential protective factor in atherogenesis [Zemplenyi et. al., 1989].The purpose of this study is to calculate coupled O_2 transport along a coronary artery, in particular, the variation of partial pressure of oxygen, Po_2, along the endothelium, $Po_{2,w}$, $Po_{2,min}$, and O_2 flux from the lumen, j_{w_l}. The effect of avascular wall thickness, δ, on $Po_{2,min}$ is studied over a range of blood mean flow rates, Q, from basal to hyperemic flows.

METHODS

The analysis is for an axi-symmetric model of coronary artery of diameter d_e = 3 mm and an axial length of 7 cm. The thickness of avascular wall is varied from 200 to 300 μm. Branching and curvature effects are not taken into account

The species transport equation in the lumen is:

$$(1+\phi')(Dc/Dt) = D_b \nabla^2 c \qquad (1)$$

and that in the avascular wall region is:

$$(Dc/Dt) = D_w \nabla^2 c - w \qquad (2)$$

where c is the oxygen concentration in ml_o/ml_{blood}. It is related to Po_2 as $c = (\alpha \cdot Po_2)$, where the value of the solubility coefficients in blood, α_b, and in the wall, α_w, are 3×10^{-5} ml_o/ml-mmHg. The quantity ϕ' introduces nonlinearity in the convective transport in the lumen side equation since it is related to the slope of the nonlinear hemoglobin saturation curve by the relation

$$\phi' = ([H]/\alpha)(\partial S/\partial Po_2) \qquad (3)$$

where, $[H] = 0.2\ ml_o/ml_{blood}$ is total oxygen carrying capacity of hemoglobin in blood and S is the saturation function. Value of O_2 consumption rate in the avascular wall, w, is 1.3×10^{-4} ml_o/ml_{tissue}-s as measured by Crawford et. al. [1983] with a microcathode in normal dog femoral arteries, which are roughly similar in size to human coronary vessels. The blood and wall density is 1.05 gm/cm^3, while diffusivity of oxygen in blood and in the wall is 1.0×10^{-5} cm^2/sec. The Carreau model for shear rate thinning non-Newtonian viscosity, having infinite shear rate viscosity of 3.45 cP, is used for the blood.

For velocity field calculations, constant flow rates of 50 (basal), 100 and 180 ml/min (hyperemic), with fully developed parabolic profile for velocity, are applied at inlet. No slip boundary condition is applied at the lumen-wall interface. Reynolds number, Re_e, based on d_e, range from 100 to 360 and Peclet number, Pe, range from 3.45×10^5 to 12.4×10^5. The Schmidt number Sc is 3450. For concentration field calculations, uniform concentration of O_2, corresponding to normal blood Po_2 of 95 mmHg is applied at inlet and at vasa vasorum O_2 concentration corresponding to $Po_{2,V}$ of 45 mmHg is applied. Zero flux boundary condition is applied at the axis. The oxygen transport from the blood in the lumen to the wall has continuity of flux across the endothelial wall. Galerkin finite element method is used for the calculations [Moore and Ethier, 1997]. The computations with the wall as porous media with a filtrate velocity of $4x10^{-6}$ cm/s show less than 1% difference in Po_2 values as compared to the solid wall. Hence, the wall region is modeled as a solid.

RESULTS AND DISCUSSION

Figure 1 shows radial variation of Po_2 at axial position, z, of 7 cm from inlet and δ = 300 μm for basal to hyperemic flows. O_2 concentration boundary layer of ~80 μm is observed in the lumen. The Po_2 decreases near the endothelial wall and the gradient causes transport of O_2 from the lumen blood to inside avascular region of the wall. The Po_2 reaches minimum value, $Po_{2,\min}$, in the wall where the radial flux becomes zero. Subsequent to the minima, the Po_2 increases radially outward towards vasa vasorum as it acts as an additional source of O_2.

Figure 2 shows the variation of the $Po_{2,w}$ with axial length for avascular thickness of δ = 300 μm from basal to hyperemic flows. $Po_{2,w}$ decreases along the length because of the consumption of O_2 along the wall. The $Po_{2,w}$ is lower for basal flow because of lower O_2 flux to the endothelial wall. This is due to the combined effect of the constant O_2 consumption in the wall and relatively thick O_2 boundary layer in the lumen even though the concentration gradient is higher for basal flow. The figure 3 shows variation of $Po_{2,\min}$ with wall thickness δ for different flow rates. At δ = 275 μm the $Po_{2,\min}$ value reaches ~15 mmHg, consistent with the findings of Zemplenyi et. al. [1989].

Table 1 shows the effect of avascular thickness on various parameters such as $Po_{2,\min}$, $Po_{2,w}$, δ_b, j_{w_l} , the mass transfer coefficient h and Sherwood number. The avascular thickness has significant effect on the Po_2 profiles in the wall. For δ = 300 μm, and Q = 50 ml/min the $Po_{2,\min}$ reaches lowest value of 4.7 mmHg as compared to 33.6 mmHg for δ = 200 μm because of the higher consumption in the wall. The change of wall thickness does not have any effect on δ_b and the change of flow rates does not have significant effects on $Po_{2,\min}$. Flux to the wall, j_{w_l} , is affected relatively more by the wall thickness compared to the flow rate. j_{w_l} increases by ~25% for δ = 300 μm compared with that for δ = 200 μm at same flow rate. It increases by ~5% for the hyperemic flow compared to the basal flow for the same wall thickness. The flow rate affects the O_2 concentration boundary layer significantly. These results for the unstenosed artery form the fundamental basis for future calculations in stenosed arteries, which will also be presented at the conference.

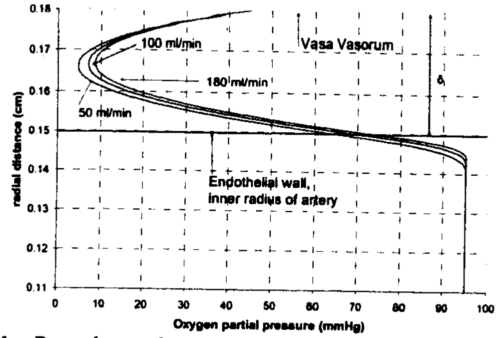

Fig.1: Po_2 along the radial distance for Q = 50, 100, 180 ml/min, δ = 300 μm at axial distance z = 7 cm.

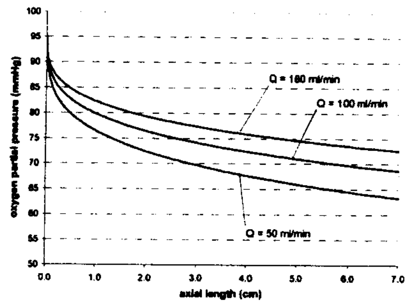

Fig. 2: Variation of $Po_{2,w}$ with axial length. δ = 300 μm

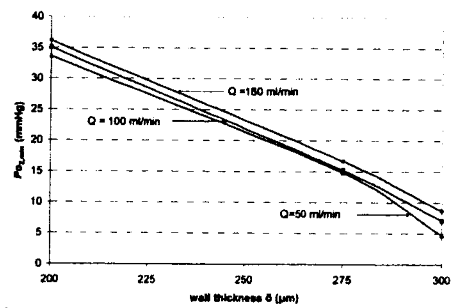

Fig. 3: Variation of $Po_{2,\min}$ for variable δ at z = 7 cm

Table 1: Effect of wall thickness on various parameters

Q ml/min	$Po_{2,min}$ mmHg	$Po_{2,w}$ mmHg	δ_b μm	j_{w_l} ml_o/ml-sec	h cm/sec	Sh
δ = 200 μm, z =7 cm						
50	33.6	69.5	80	1.71e-6	2.23e-3	66.9
100	35.0	73.7	60	1.78e-6	2.78e-3	83.4
180	36.0	77.0	50	1.83e-6	3.38e-3	101
δ = 300 μm, z = 7cm						
50	4.7	63.3	80	2.20e-6	2.31e-3	69.3
100	7.1	68.6	60	2.26e-6	2.85e-3	85.5
180	9.0	72.6	50	2.30e-6	3.42e-3	103

REFERENCES

Crawford, D.W., Blankenhorn, D.H. 1991, "Arterial wall Oxygenation, Oxyradicals, and Atherosclerosis", Atherosclerosis, 89, pp 97-108.

Zemplenyi, T., Crawford, D.W., Cole, M.A., 1989, "Adaptation to arterial wall Hypoxia Demonstrated in vivo With microcathodes", Atherosclerosis, 76, pp 173-179.

Crawford, D.W., Cole, M.A., Back, L.H., 1983, "Evidence for the blood oxygen boundary layer Po_2 gradient (ΔPo_2) as a significant determinant of intimal ($Po_{2,w}$) and lowest medial Po_2 ($P_{\delta n}$) in the in vivo dog femoral artery". Oxygen transport to tissue, IV, Eds. Bicher, H.I. and Bruley, D.F., Plenum Pub. Corp., New York, pp 197-209.

Moore, J.A., Ethier, C.R., 1997, "Oxygen mass transfer calculations in large arteries" ASME J. Biomech. Eng., 119, pp 469-475.