Proceedings of IMECE'03
2003 ASME International Mechanical Engineering Congress & Exposition
Washington, D.C., November 16-21, 2003
IMECE2003-43145

Pressure-Flow Characteristics during Peristaltic Transport of non-Newtonian Fluid in Distensible Tube with Different Wave Forms

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ABSTRACT

This study analyzes the pressure-flow characteristics during the peristaltic pumping of power law fluids in an axi-symmetric non-uniform distensible tube. The analyzed geometry is of a diverging shape that is common in several biological flow conduits, especially in mammals. Using the Fourier series, the dimensionless wall co-ordinates for sinusoidal, triangular, trapezoidal, and square wave forms are obtained to simulate wall movement. Equations expressing the pressure–flow rate relationship for different wall shapes are developed from the wave equation. Pressure-flow and velocity plots are obtained by solving the equations numerically. The results clearly invalidate the assumption of the fluid as Newtonian and demonstrate that a maximum flow rate can be achieved when the wall movement follows a square wave pattern.

INTRODUCTION

Research on the peristaltic transport is motivated by interest in the physiological fluid transport in many biological systems, e.g. urine transport from kidney to the bladder through the ureter, movement of chime, and transport of spermatozoa in the vas deferens. The initial study conducted by Jaffrin and Shapiro (1971) introduced the lubrication theory model in which the effects of fluid inertia and wall curvature are neglected. Pozrikidis (1987) extended the same theory for a two dimensional channel to analyze the effect of adverse pressure gradient on the peristaltic flow and compared it with the pure peristaltic flow. Eytan and Elad (2001) in their publication discussed in detail the phenomenon of trapping and reflux during the intrauterine fluid flow.

Most of the previous studies have been carried out by assuming the fluid to be Newtonian and the waveform to be sinusoidal. A food bolus passing through the esophagus or the movement of spermatozoa in the ductus efferentes may be suitably modeled by considering a non-Newtonian fluid. A limited number of literatures are available for non-Newtonian fluid analysis of the peristaltic flow. In the present research, an attempt is made to study the peristaltic transport in divergent tubes with different wave forms, flow behavior index, and amplitude ratio with specific application to the flow of spermatic fluid in the vas-deferens of a rhesus monkey.

ASSUMPTIONS

A power law fluid is considered to be flowing through a diverging tube (vas-deferens of a rhesus monkey) whose radius increases linearly as given by the expression $a=a_0+kx$, where a_o is the inlet radius of the tube in m and k is a constant. Since the Re is very low, the inertia term in the momentum equation can be neglected (Banerjee, 1987). The a_0/λ ratio, λ is the wavelength of the peristaltic wave in m, is so small that the transverse velocity component can be neglected in comparison with the longitudinal values. Moreover, Li and Brasseur (1993) illustrated that the effect of non-integral wave form is negligible and consequently the ratio of tube length to the wavelength can be assumed to be an integer. Waves of different shapes are assumed to travel from left to right at a speed of c.

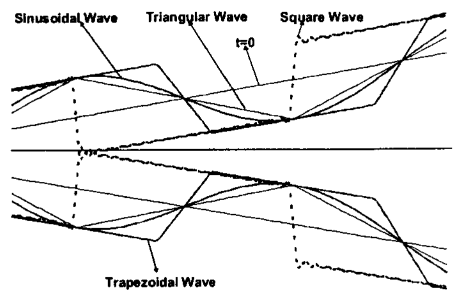

Figure 1: Flow Geometry for different wave forms

PROBLEM FORMULATION

All variables are non-dimensionalised and measured in laboratory frame of reference, unless specified.

Governing Equations and Boundary Conditions

By applying the long wavelength approximation and neglecting inertia terms, the appropriate equation of motion after introducing non-dimensional variables reduces to

$$\frac{\partial p}{\partial x}=-\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(-\frac{\partial u}{\partial r}\right)^{n}\right];\ \frac{\partial p}{\partial r}=0 \tag{1}$$

where p is the pressure, r is the radial distance, x is the axial distance, u is the axial velocity, and n is the flow behavior index.

The boundary conditions are:

$$\partial u/\partial r=0 \quad \text{at} \quad r=0 \tag{2}$$

$$u=0 \quad \text{at} \quad r=h=1+(\lambda kx/a_0)+\Phi(WaveForm) \tag{3}$$

where h is the wall co-ordinate, t is the time, and Φ is the amplitude ratio.

From the Fourier series, the wave equation for sinusoidal wave form is obtained as

$$h=1+(\lambda kx/a_0)+\Phi\sin 2\pi(x-t) \tag{4}$$

An expression for axial velocity is obtained by solving the governing equation for u and applying the boundary conditions as

$$u(x,r,t)=n(-0.5\,(dp/dx))^{1/n}\,(1/(1+n))[h^{(1+n/n)}-r^{(1+n/n)}] \tag{5}$$

The instantaneous volume flow rate in laboratory frame of reference is given by

$$Q(x,t)=(n\pi/3n+1)(-0.5\,(dp/dx))^{1/n}\,h^{(3n+1)/n} \tag{6}$$

where Q is the volume rate of flow.

Integrating the above equation over the length of the tube, we get

$$(n/(3n+1))^n \Delta P_L(t) = -2\int_0^{L/\lambda} [Q(x,t)/\pi]^n/(h^{3n+1})\, dx \qquad (7)$$

where ΔP_L is the pressure rise.

This integral is solved numerically using the trapezoidal rule and the instantaneous volume rate of flow in the laboratory frame is obtained as

$$Q(x,t)/\pi = \bar{Q}/\pi - \Phi^2/\pi + (1+(\lambda kx/a_0))2\Phi \sin 2\pi(x-t) + \Phi^2 \sin^2 2\pi(x-t) \qquad (8)$$

where $\bar{Q}$ is the time averaged volume flow rate.

RESULTS AND DISCUSSION

Figure 2 depicts the difference in the pressure rise / fall over the length of the tube for a Newtonian and a power law fluid. As the value of the power law index increases from 0 to 1 the ΔP_L value also increases and it's maximum for n=1 i.e when the fluid becomes Newtonian. It's interesting to note that the ΔP_L remains constant when the cross-section of the tube is uniform (k=0). As evident from Figure 2 amplitude ratio plays an important role in the peristaltic motion since higher amplitide ratio implies better squeezing action.

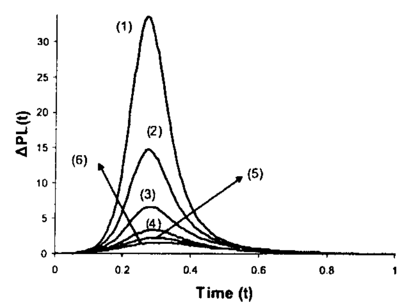

Figure 2: Pressure rise / fall through the length of the distensible tube for different power law indexes and amplitude ratios when a sinusoidal wave is travelling with a velocity c.

Figure 3 shows the linear variation of non-dimensionalised average pressure with non-dimensionalised flow rate. It can be seen that the values of maximum average pressure and the maximum flow rate at zero average pressure are highest for the square wave shape and lowest for triangular wave form.

It is evident that the square wave is more suitable for pumping action. The slope of this figure for the peristaltic flow is smaller than that for a comparable poiseuille flow since the peristaltic motion generates extra flow rate in addition to that due to the pressure gradient between the tube ends.

Moreover, if the ΔP_L drops below a particular value, the average flow rate becomes negative therby, generating a reverse flow. This net retrograde motion of fluid in a direction opposite to the wave propagation on the wall is termed as "Reflux". It can cause pathological transport of bacteria through the urinary tract to the body and this irregularity in reproductive tract can lead to infertility.

Figure 4 shows the velocity profile along the tube length at one particular instant of time (t = 0.9).The dash dotted lines in the figure clearly show the presence of reverse flow; however, in this case the net movement of the fluid is in the forward direction. Further, the change in the wall shape is superimposed on the divergent tube geometry.

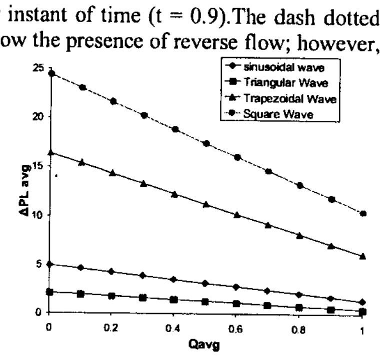

Figure 3: Pressure-Flow relationship for vas deferens of rhesus monkey at Φ =0.8 and n=1

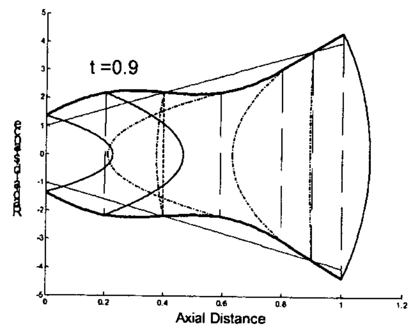

Figure 4: Velocity profile along the length of the distensible tube at t=0.9, Φ =0.8, n=1 for sinusoidal wave form

CONCLUDING REMARKS

This analysis assumed that a low Re will not affect the flow and that the radial component of velocity is negligible. Hence the inertia term is neglected and the r-direction momentum equation is neglected. However, S.Takabatake (1988) showed that in certain physiological conditions, Re does have an effect on the peristalsis. Accordingly, further studies are needed to account for the inertia term in the momentum equation as well as for improved models of non-Newtonian fluids such as Bingham or Herschel-Bulkley.

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